Solutions to exercise sheet 4

Sex, Ageing and Foraging Theory

Exercise 1: Competition for renewable resources among relatives

a. Solving for $\hat{n}(x)$ such that

$$\left.\frac{dn}{dt}\right|_{n=\hat{n}(x)} = 0, \tag{1}$$

we obtain equilibrium resource density,

$$\hat{n}(x) = \Big(1 - \frac{n_{\rm c} x}{r}\Big) K \ . \tag{2}$$

b. Substituting $\hat{n}(y_{\rm r})$ from ex. 1a above into the fitness function (eq. 3 from ex. sheet 4 together with the cost eq. 4), we find that fitness reads as

$$w(y, y_{\rm r}, x) = y\Big(1 - \frac{n_{\rm c} y_{\rm r}}{r}\Big) K - \frac{c_0}{2} y^2 \ . \tag{3}$$

Differentiating this fitness function according to the selection gradient given by eq. (5) in ex. sheet 4, we obtain

$$s(x) = \Big(1 - \frac{n_{\rm c} x}{r}\Big) K - c_0 x - R_2 \frac{n_{\rm c} x}{r} K \ . \tag{4}$$

c. Solving for x^* such that $s(x^*) = 0$, we find that the optimal strategy x^* can be written as

$$x^* = x_{\rm MSY} \frac{2Kn_{\rm c}}{c_0 r + Kn_{\rm c}(1 + R_2)} \ , \tag{5}$$

where

$$x_{\rm MSY} = \frac{1}{n_{\rm c}} \frac{r}{2} \tag{6}$$

is the foraging effort that lead to maximum sustainable yield. Eq. (5) reveals that the optimal strategy x^* decreases with relatedness, R_2, i.e. individuals evolve to forage less when they do so with relatives. In particular, even in the absence of foraging cost ($c_0 = 0$), individuals avoid over-exploitation when they forage with monozygotic twins (i.e. $x^* = x_{\rm MSY}$ when $R_2 = 1$).

Exercise 2: Risk-sensitive foraging

a. By plugging the payoff values in eqs. (7) and (8) of the ex sheet 4, we obtain

Payoff, π_i	Low condition f_{L}	High condition f_{H}
0	0.0	0.0
1	0.9	2.1
2	3.2	3.3

b. In high condition, the fecundity gain from a payoff of 1 to 2 is less than the loss from a payoff of 1 to 0. Selection should therefore favour to avoid risk in high condition individuals (i.e. $x_{\mathrm{H}} \to 0$). By contrast, the fecundity gain from a payoff of 1 to 2 when in low condition is greater than the loss from a payoff of 1 to 0. Selection should therefore lead individuals in low condition to take risk ($x_{\mathrm{L}} \to 1$).

c. The predictions made in 2b above are borne out when running individual based simulations (Fig.1).

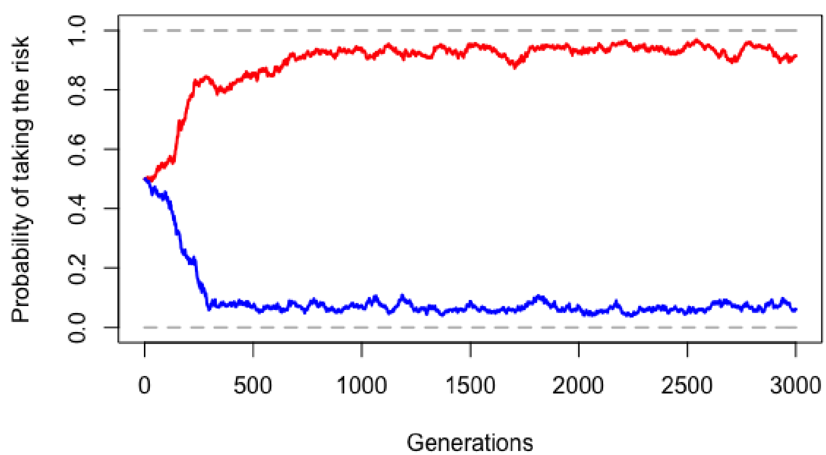

Figure 1: Evolution of the average probabilities of choosing the risk-taking strategy when in low and high condition, x_{L} (in red) and x_{H} (in blue). The population is initially monomorphic for $x_{\mathrm{H}} = x_{\mathrm{L}} = 0.5$.