

Part III - Foraging theory

Sex, Ageing and Foraging Theory

resources

energy

offspring

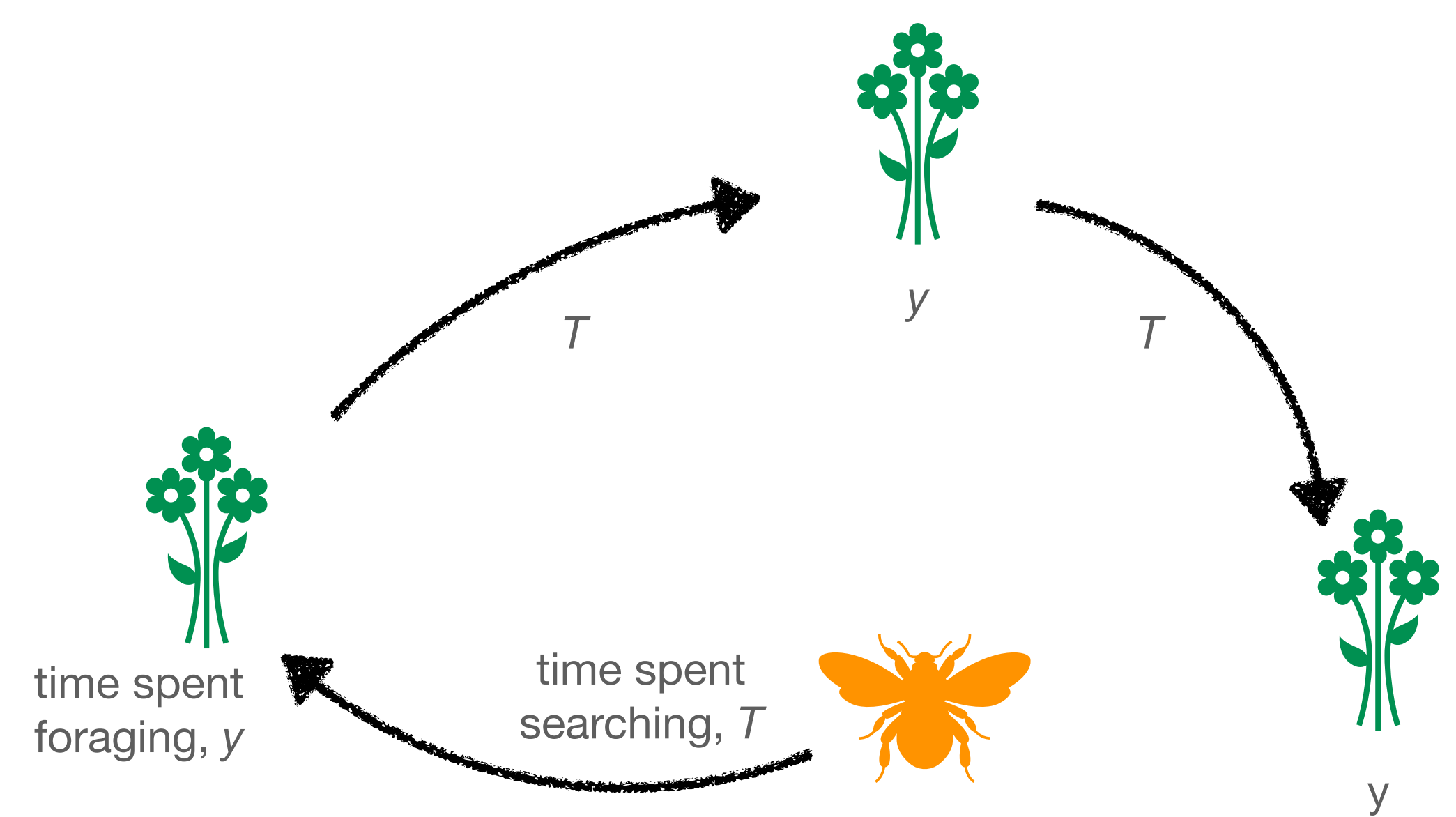
fitness



Foraging in patches

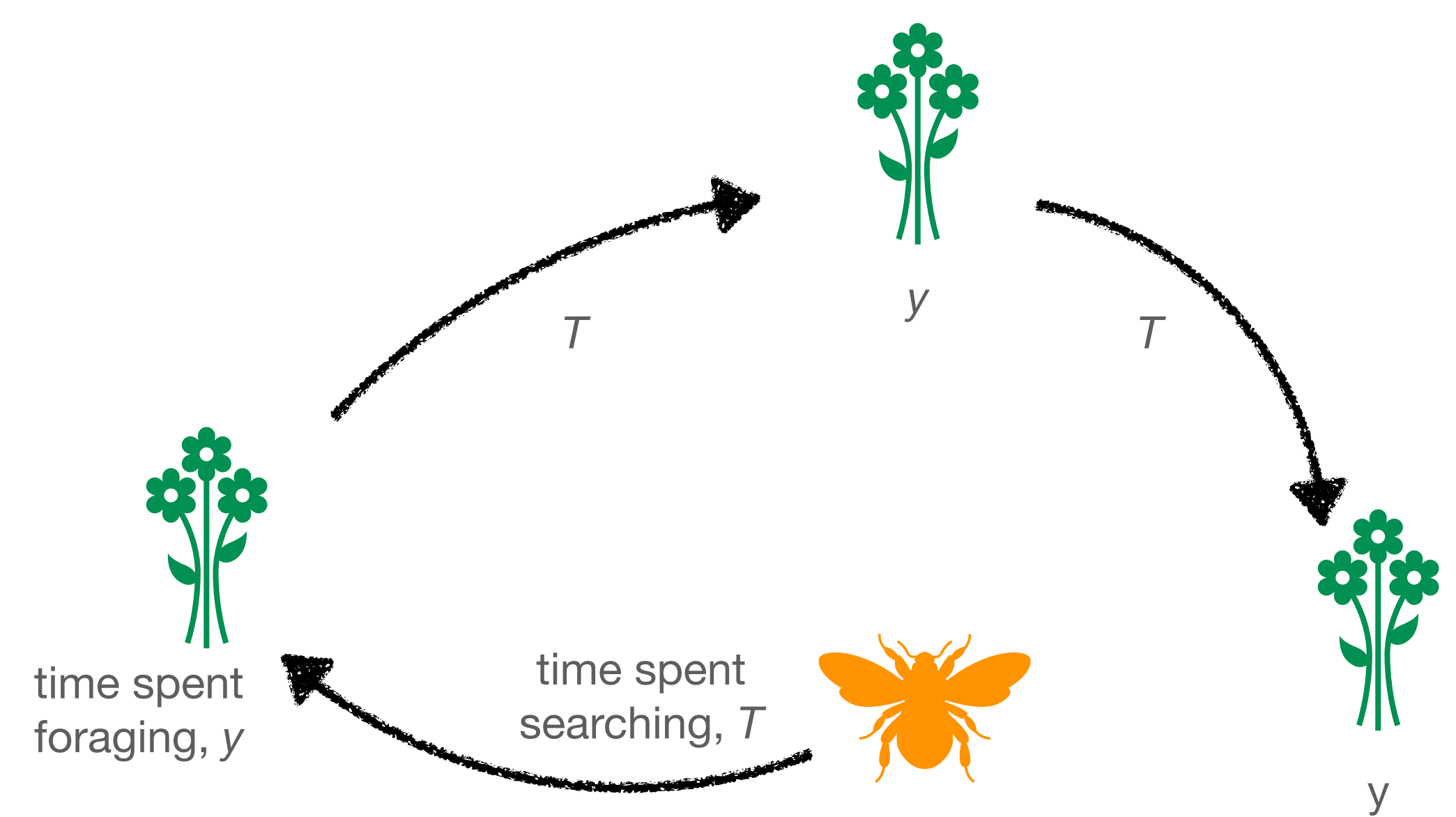
Foraging in patches

- Animal forages on multiple equivalent patches with finite amount of resources.



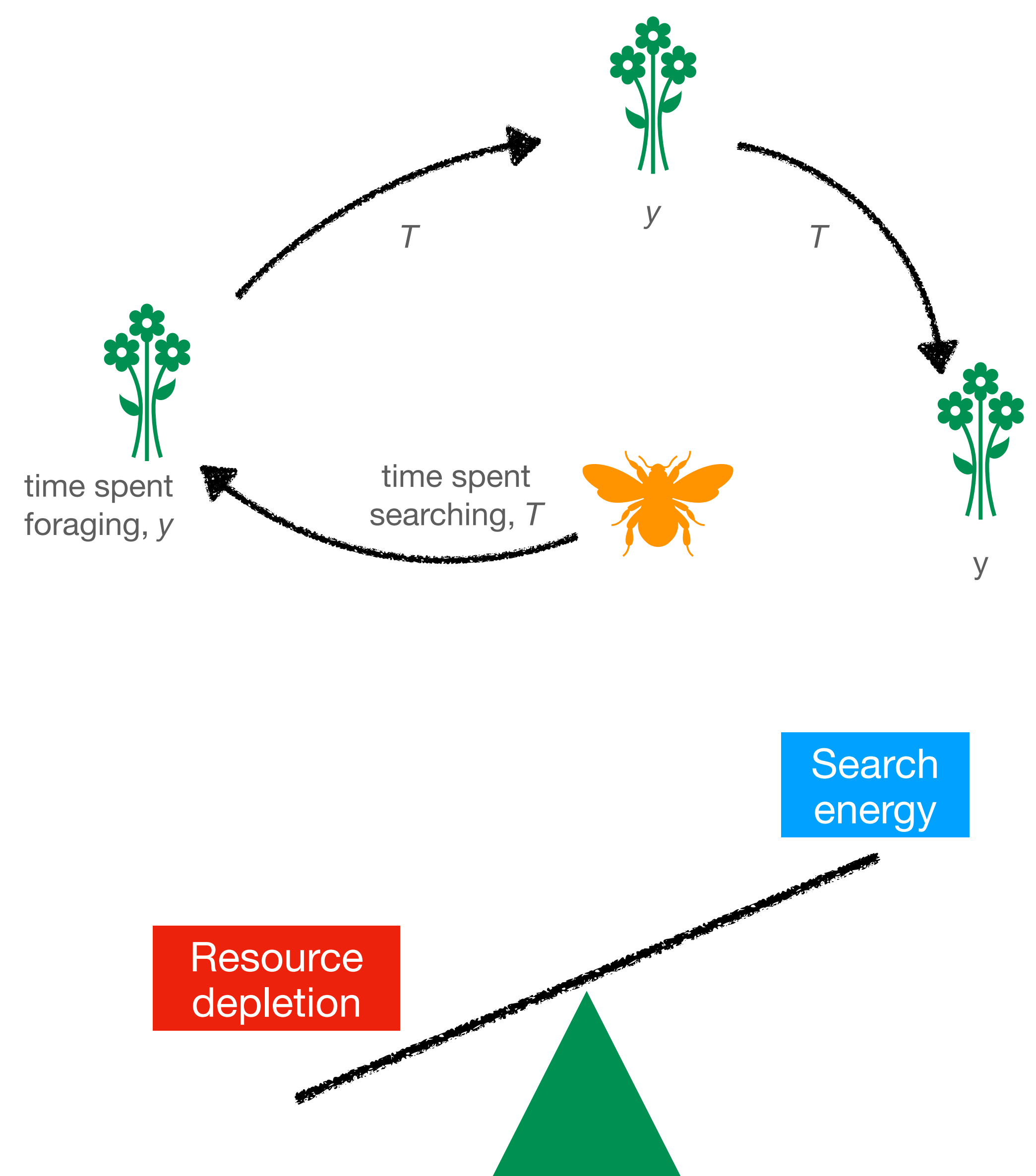
Foraging in patches

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- How much time y should it spent foraging on a single patch when searching is costly?



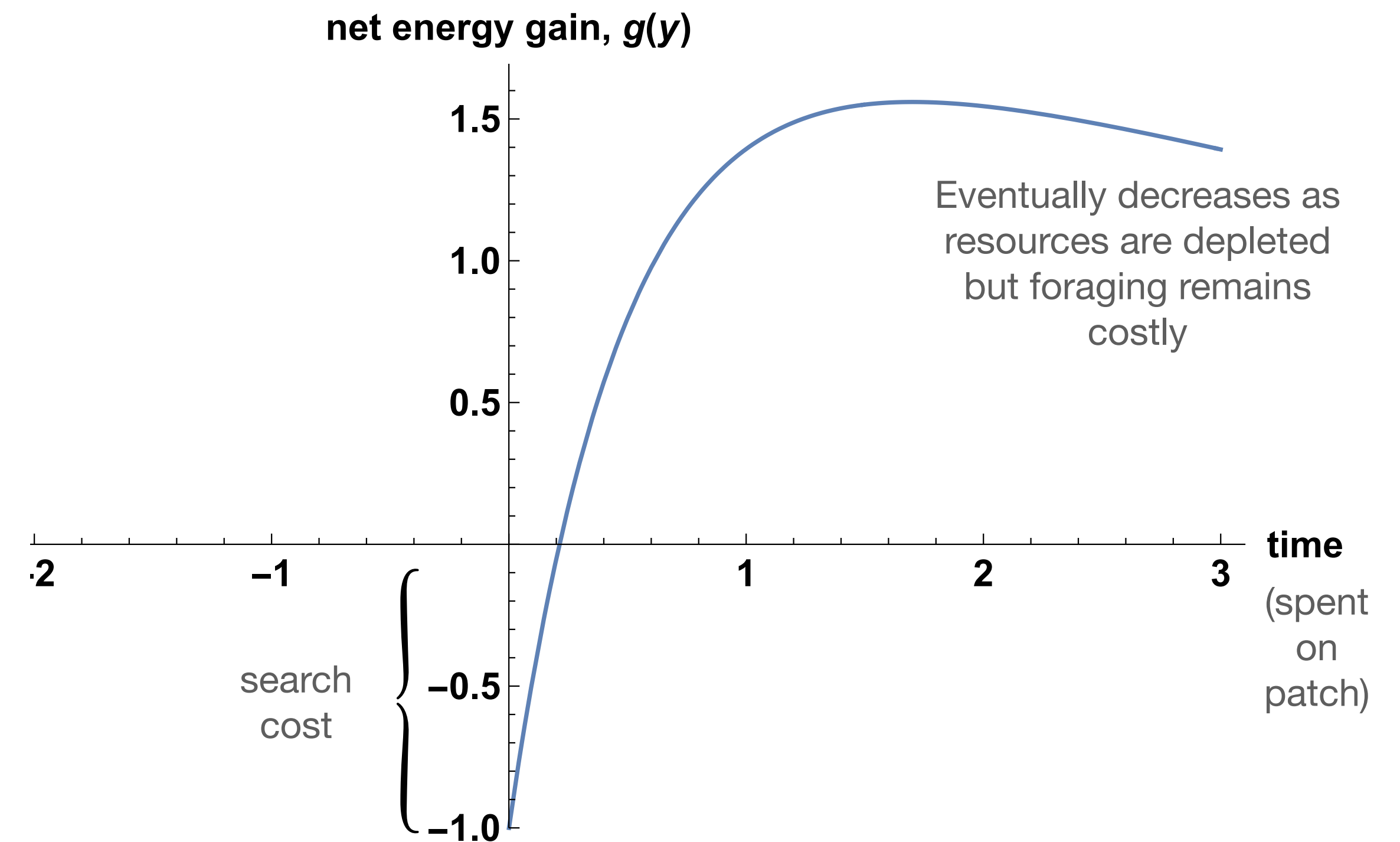
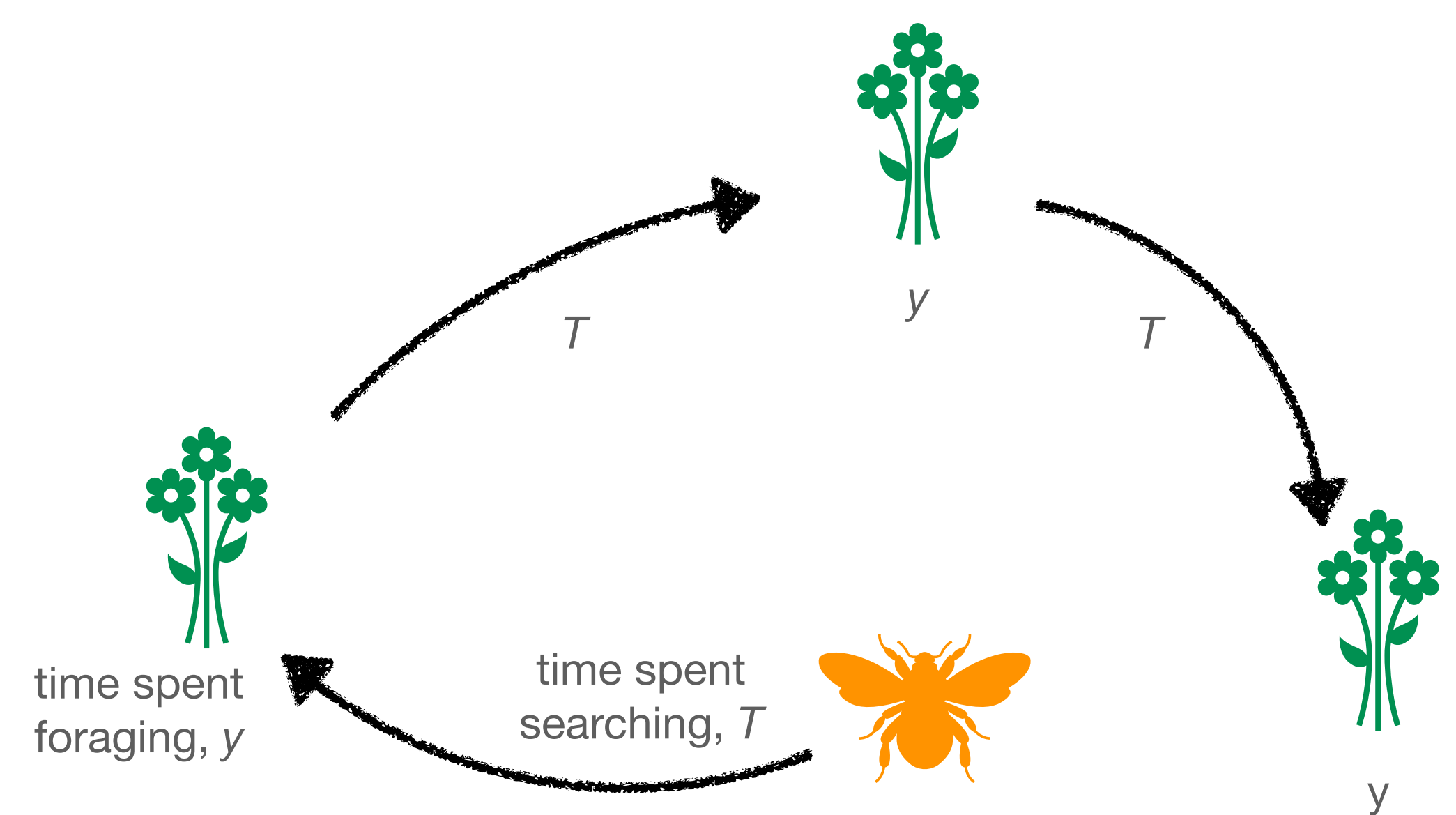
Foraging in patches

- Animal forages on multiple equivalent patches with finite amount of resources.
- How much time y should it spent foraging on a single patch when searching is costly?
- If it stays too long, resources get depleted; too short and it does not regain energy lost from search.



Energy gains

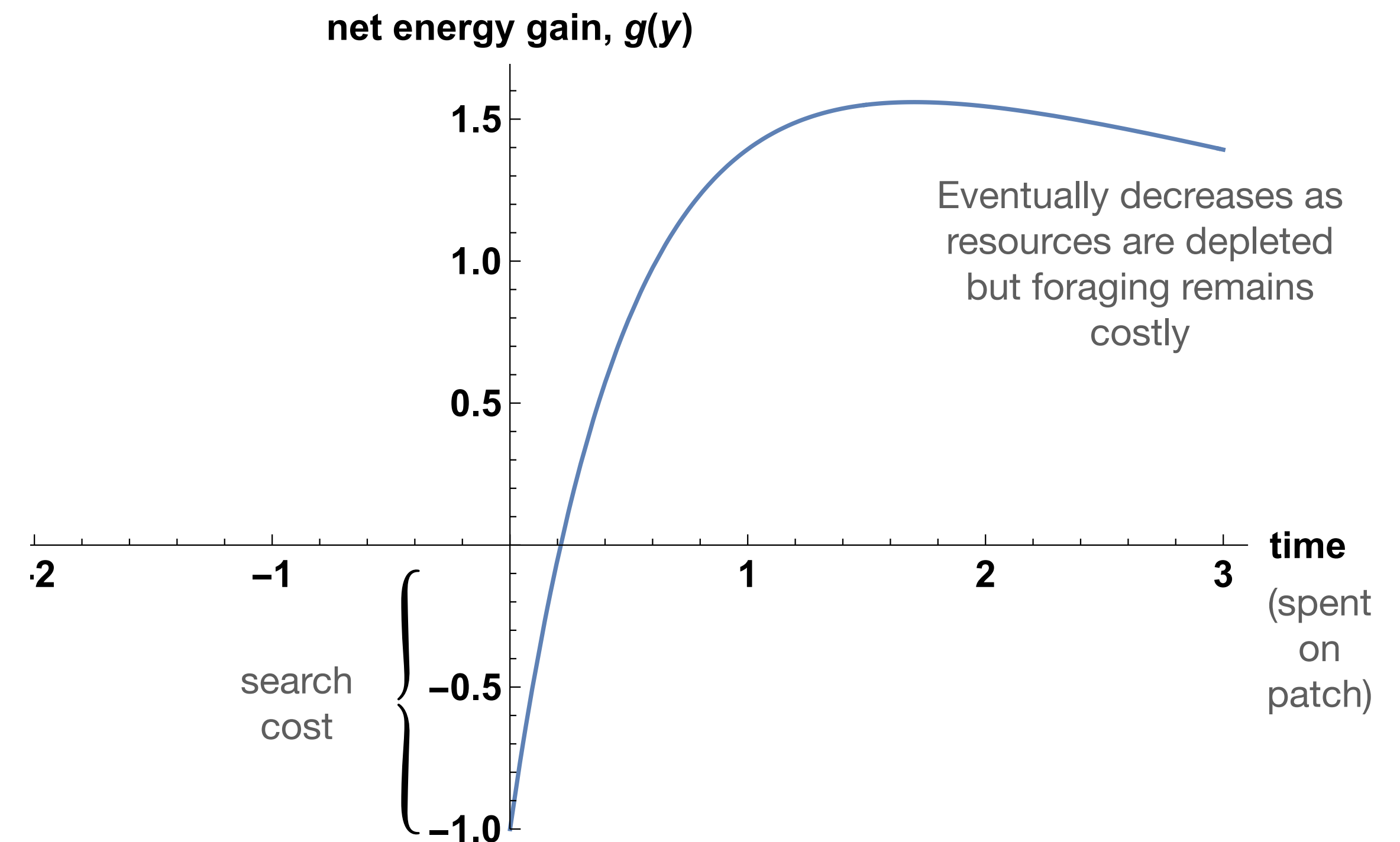
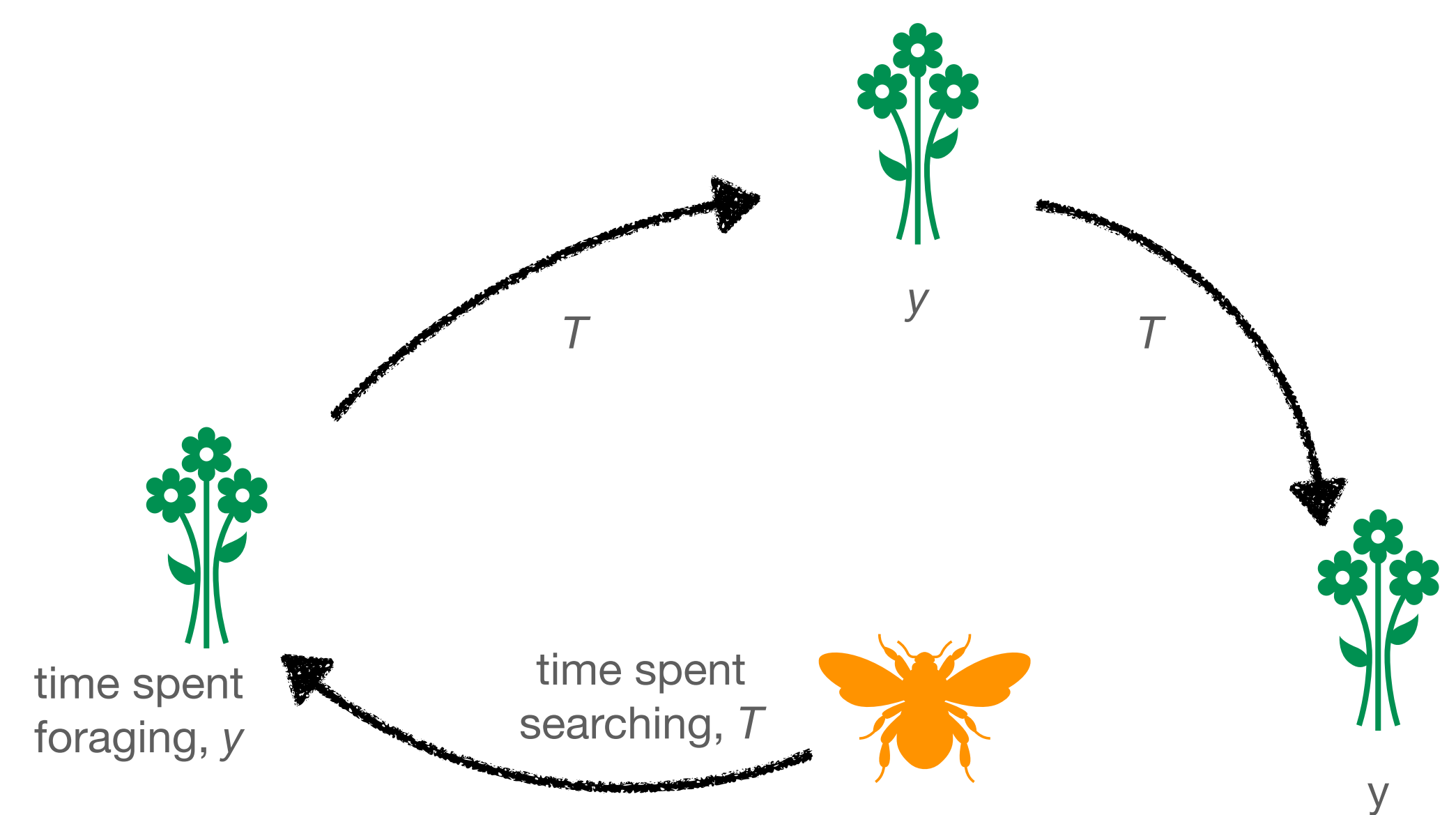
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Energy gains

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- Rate of energy gain from search + foraging :

$$R(y) = \frac{g(y)}{y + T}$$

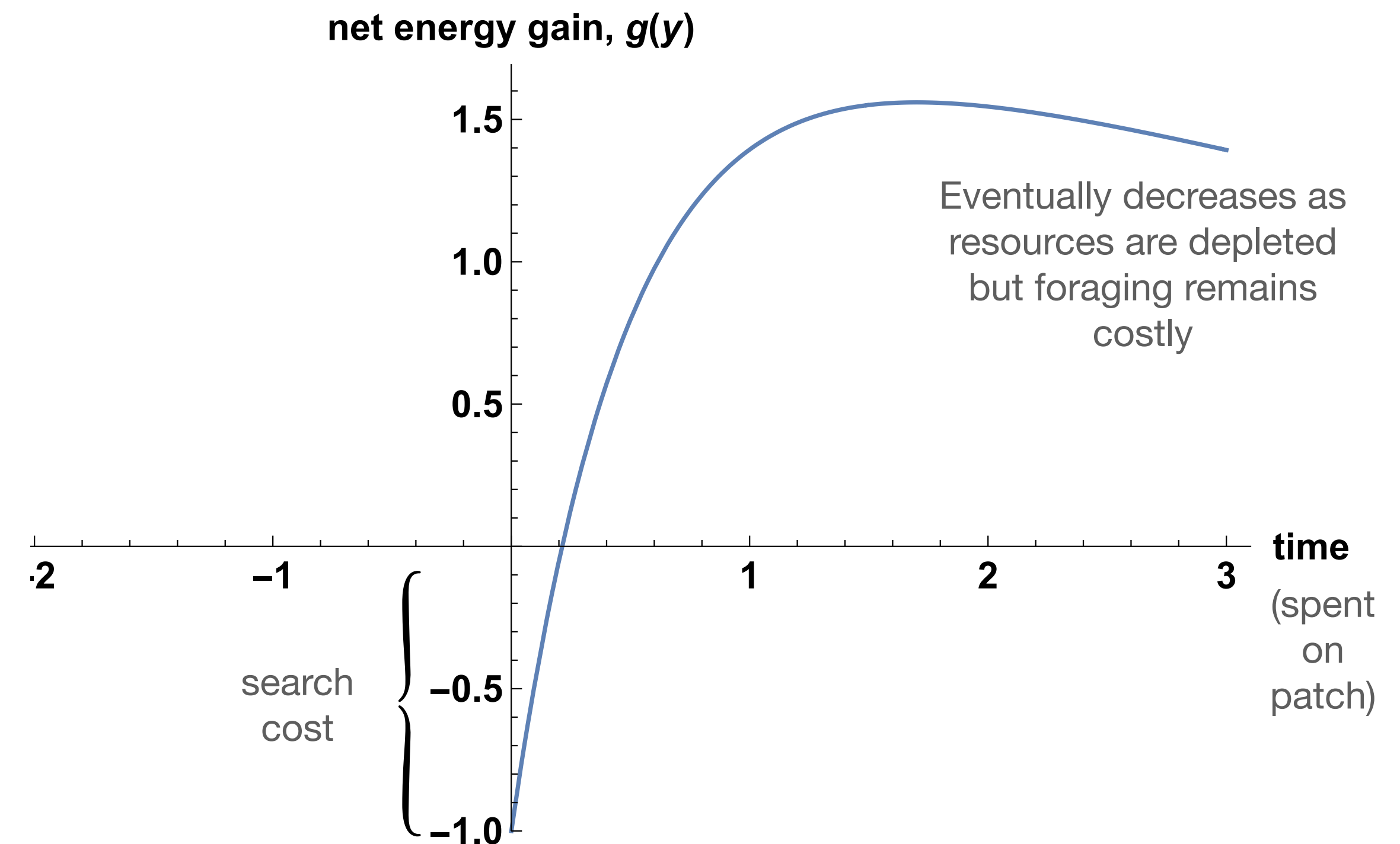
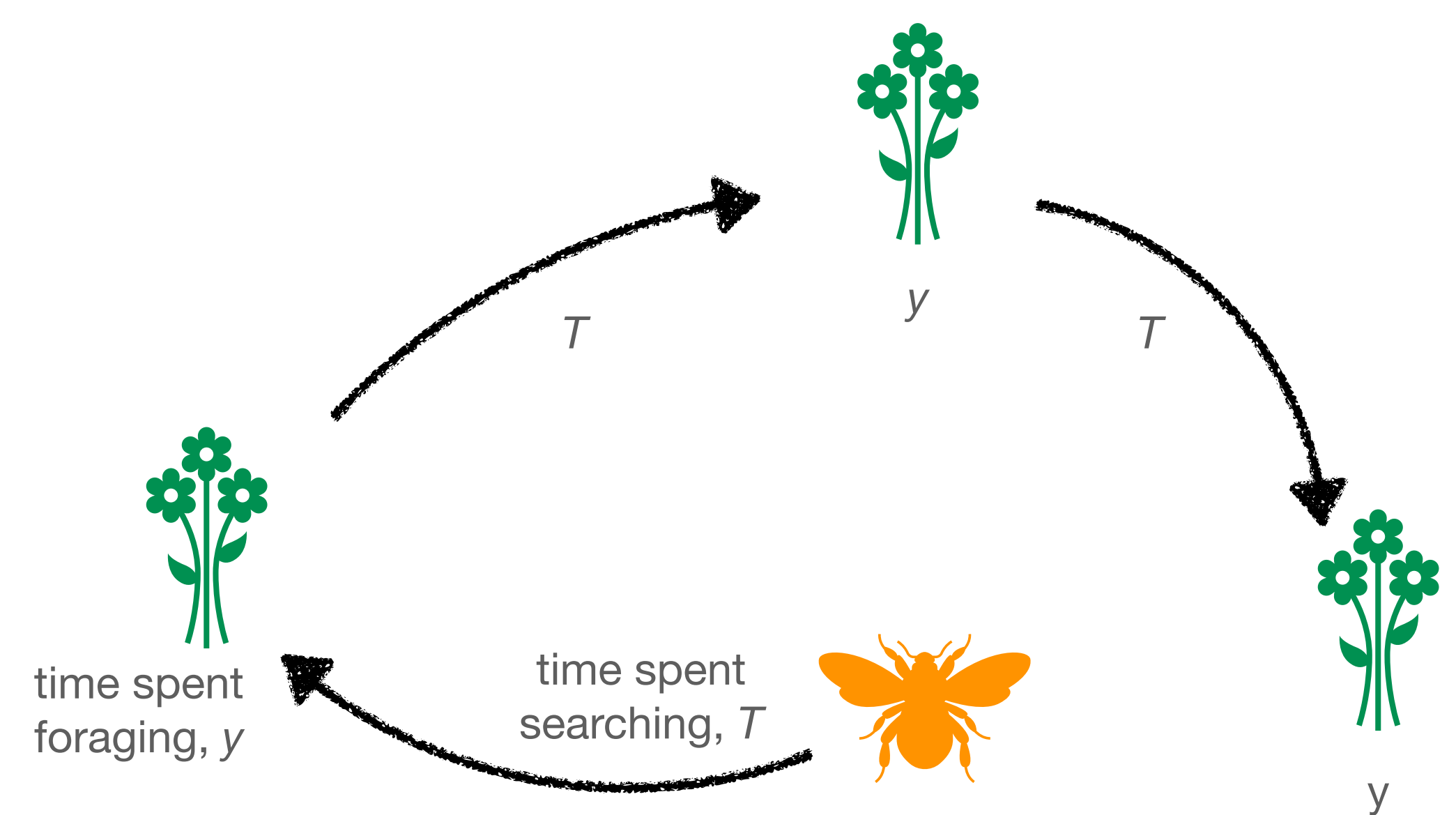


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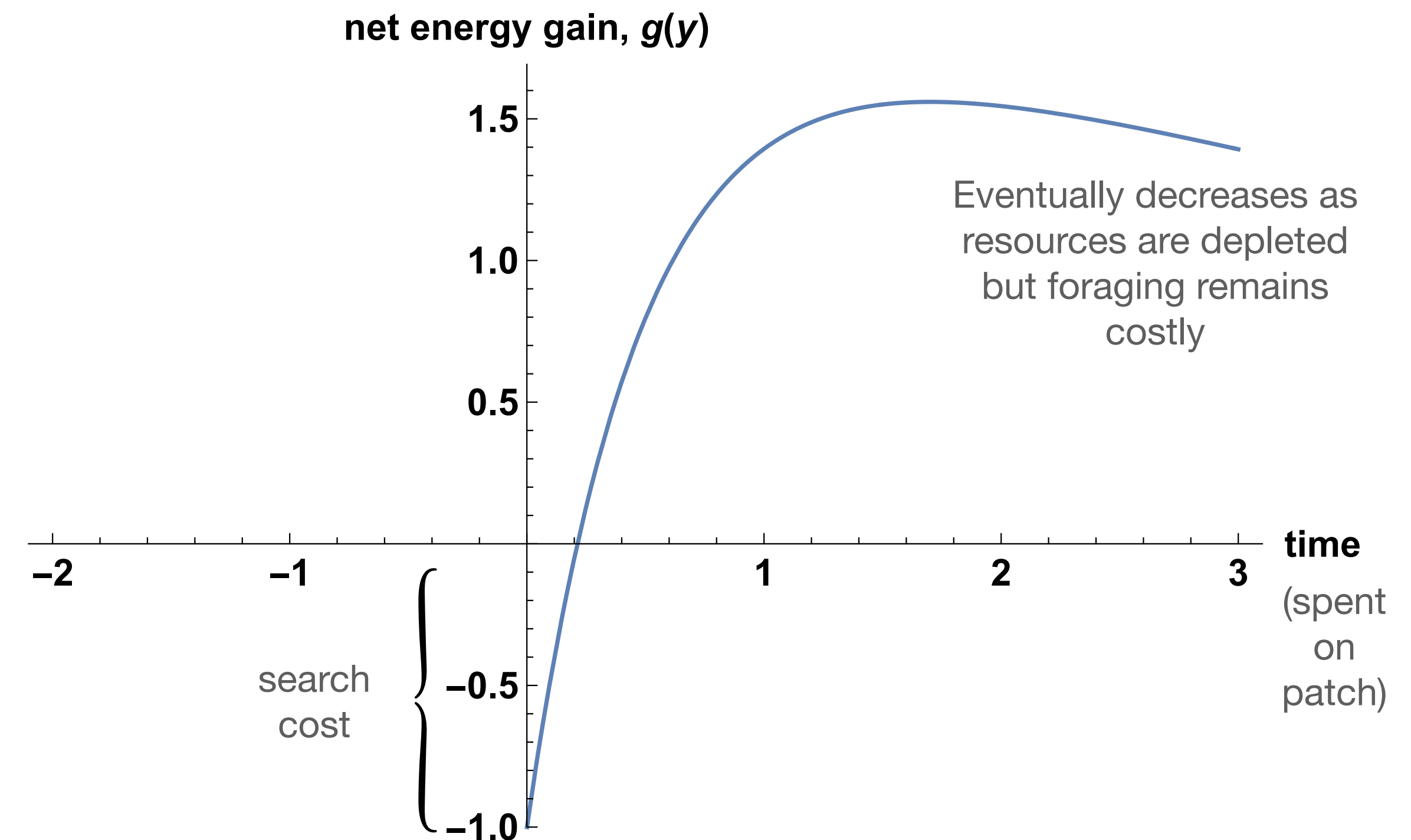
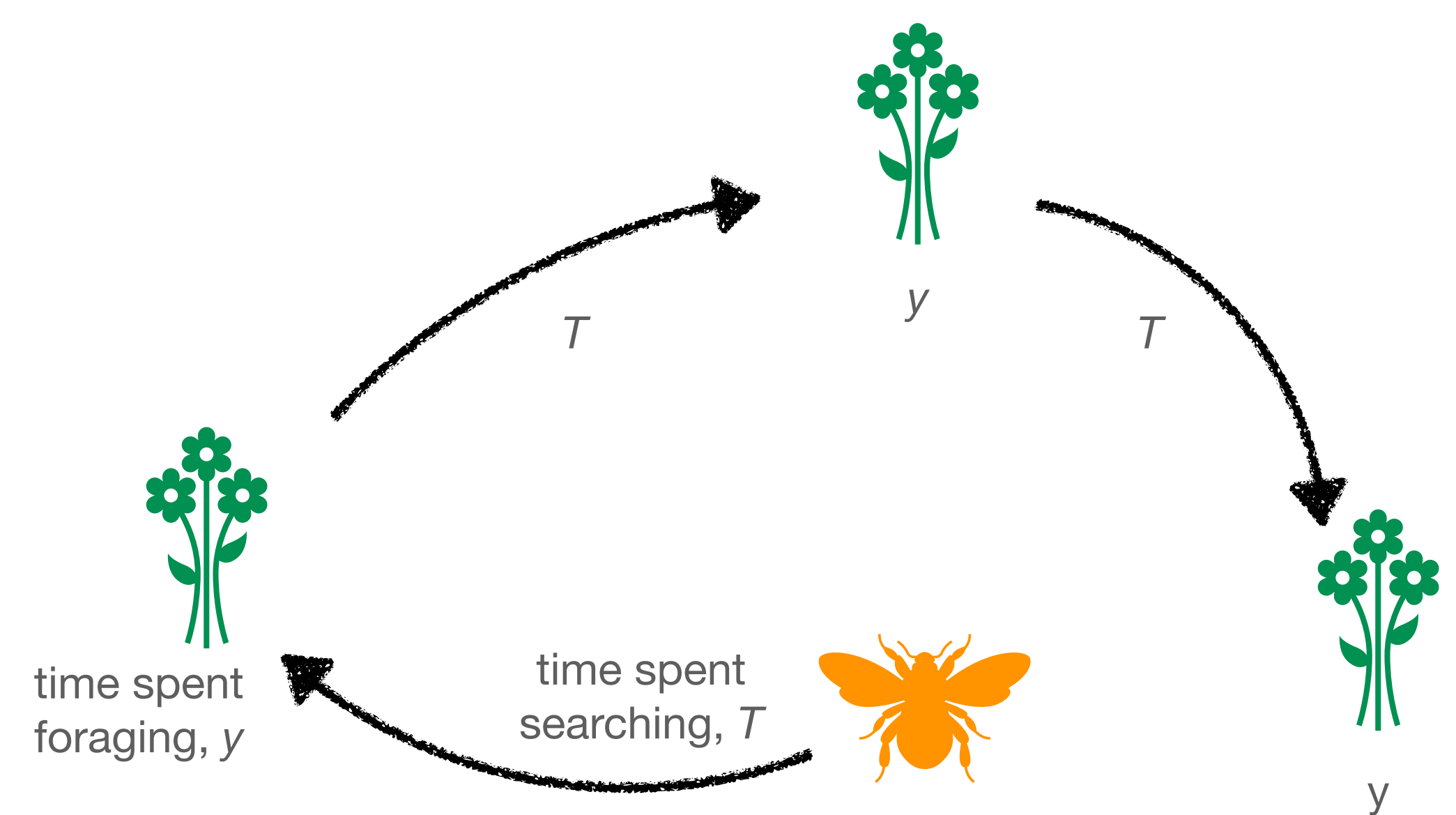
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- Selection gradient :

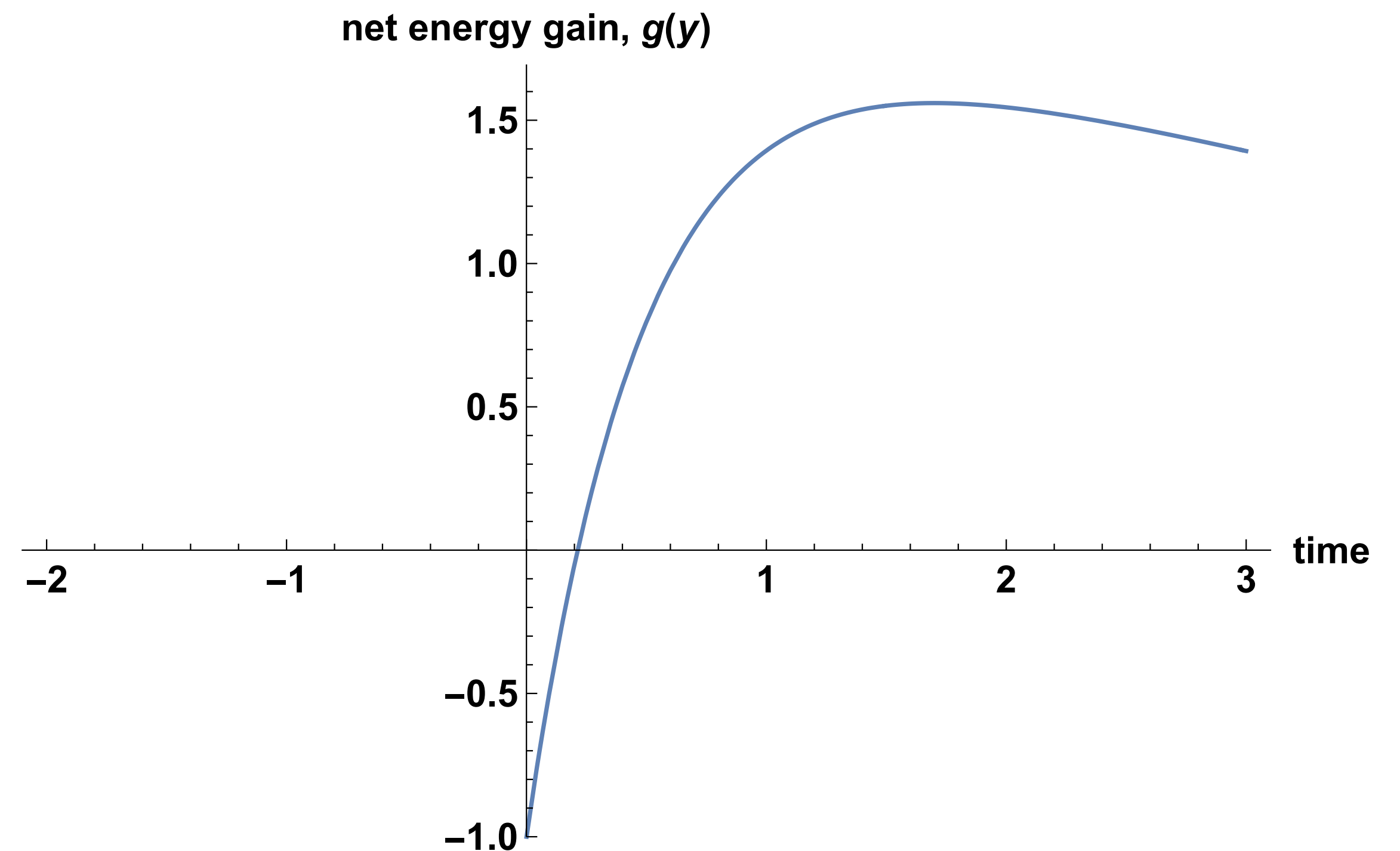
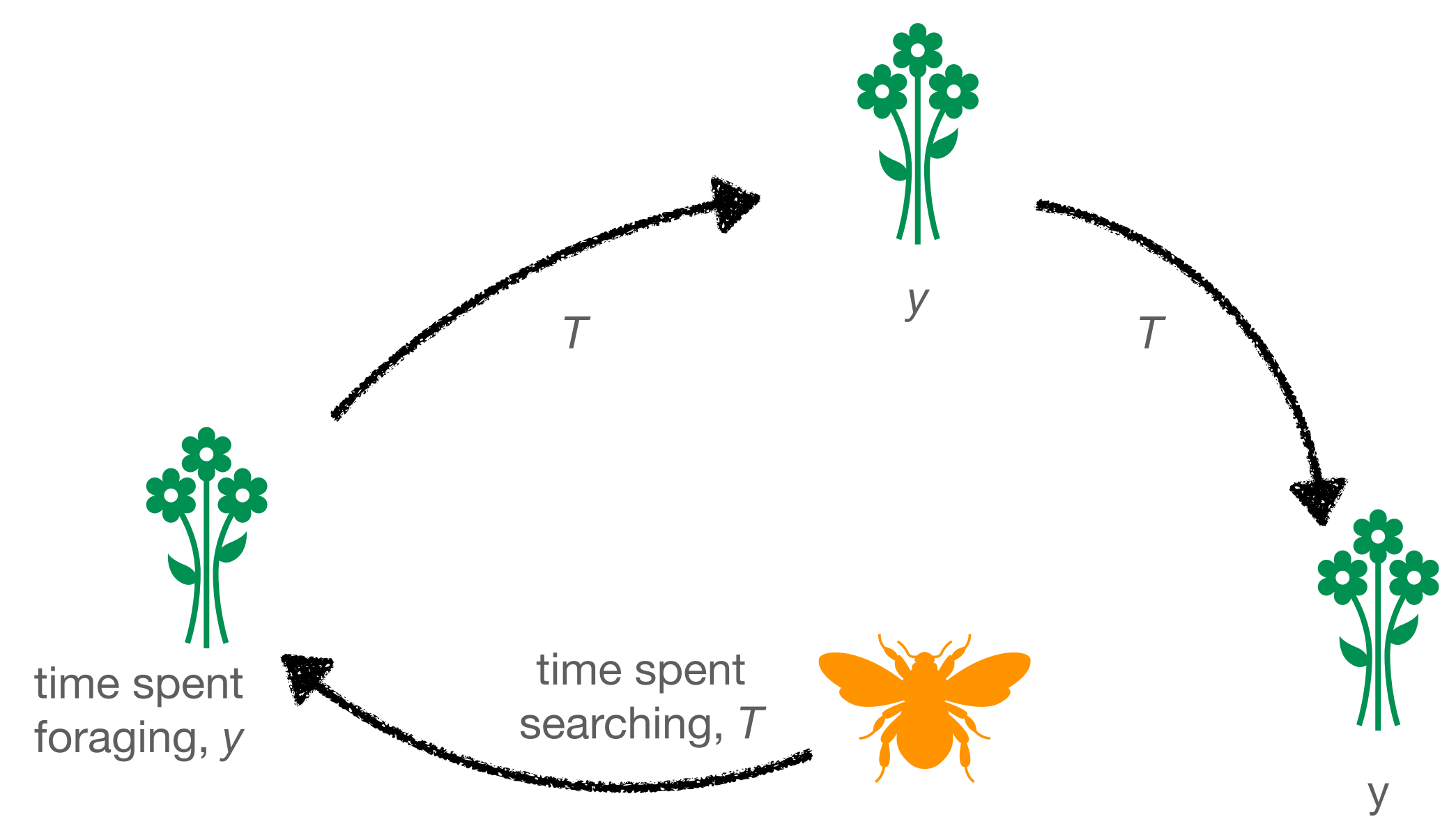
$$s(x) \propto \frac{g'(x)}{x + T} - \frac{g(x)}{(x + T)^2}$$



Marginal value theorem

Optimum x^* such that $s(x^*) = 0$,
i.e., such that

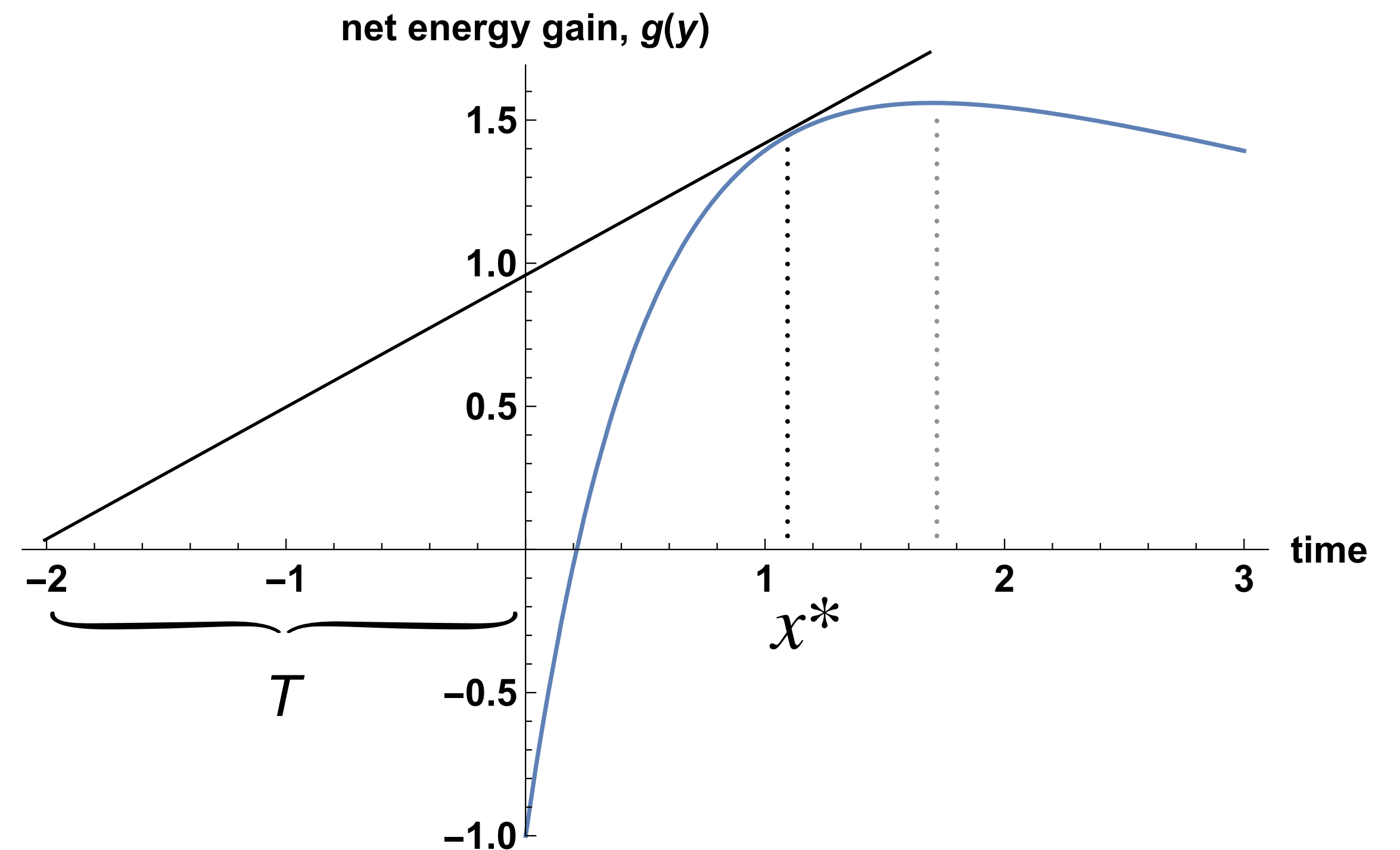
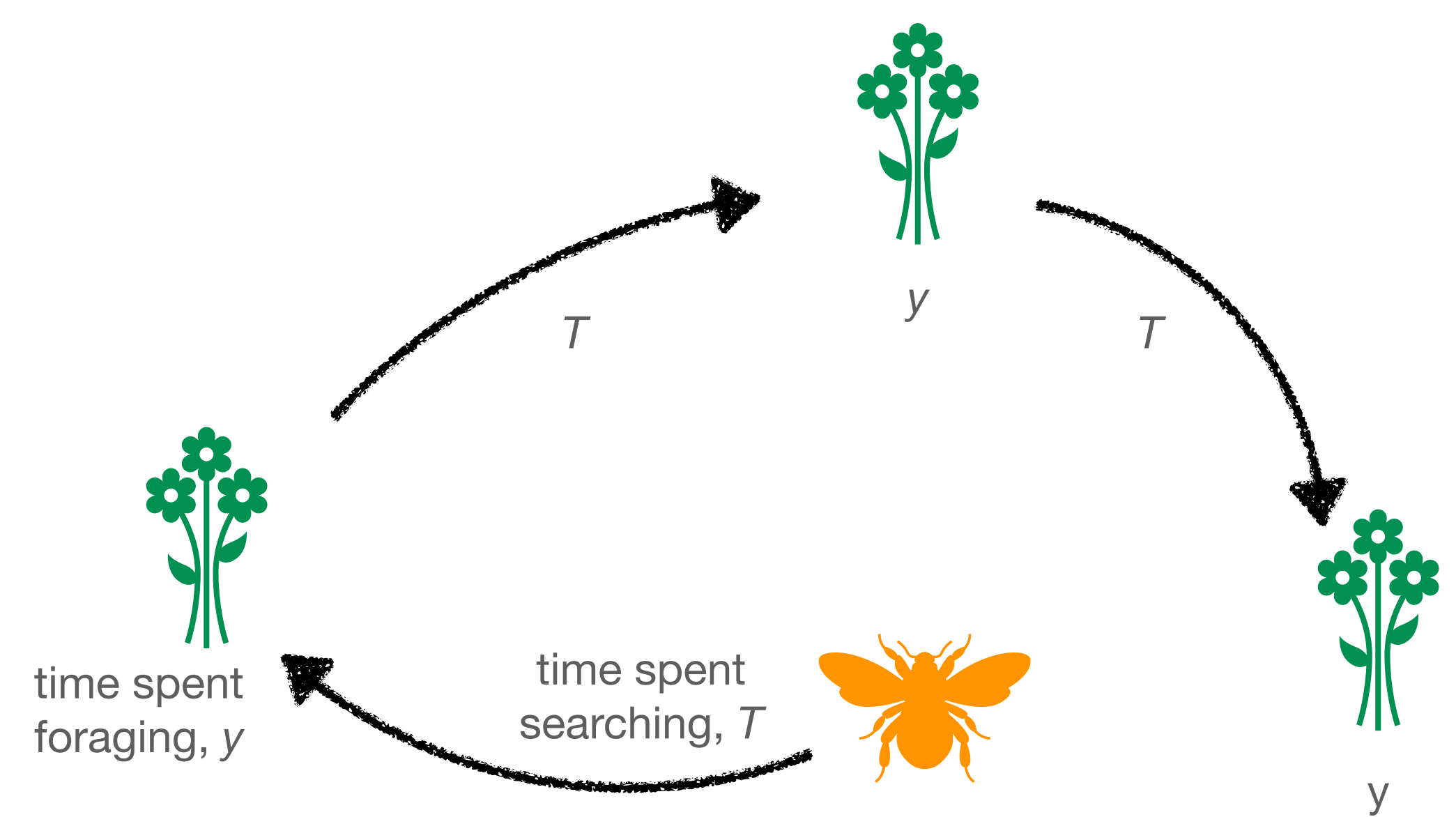
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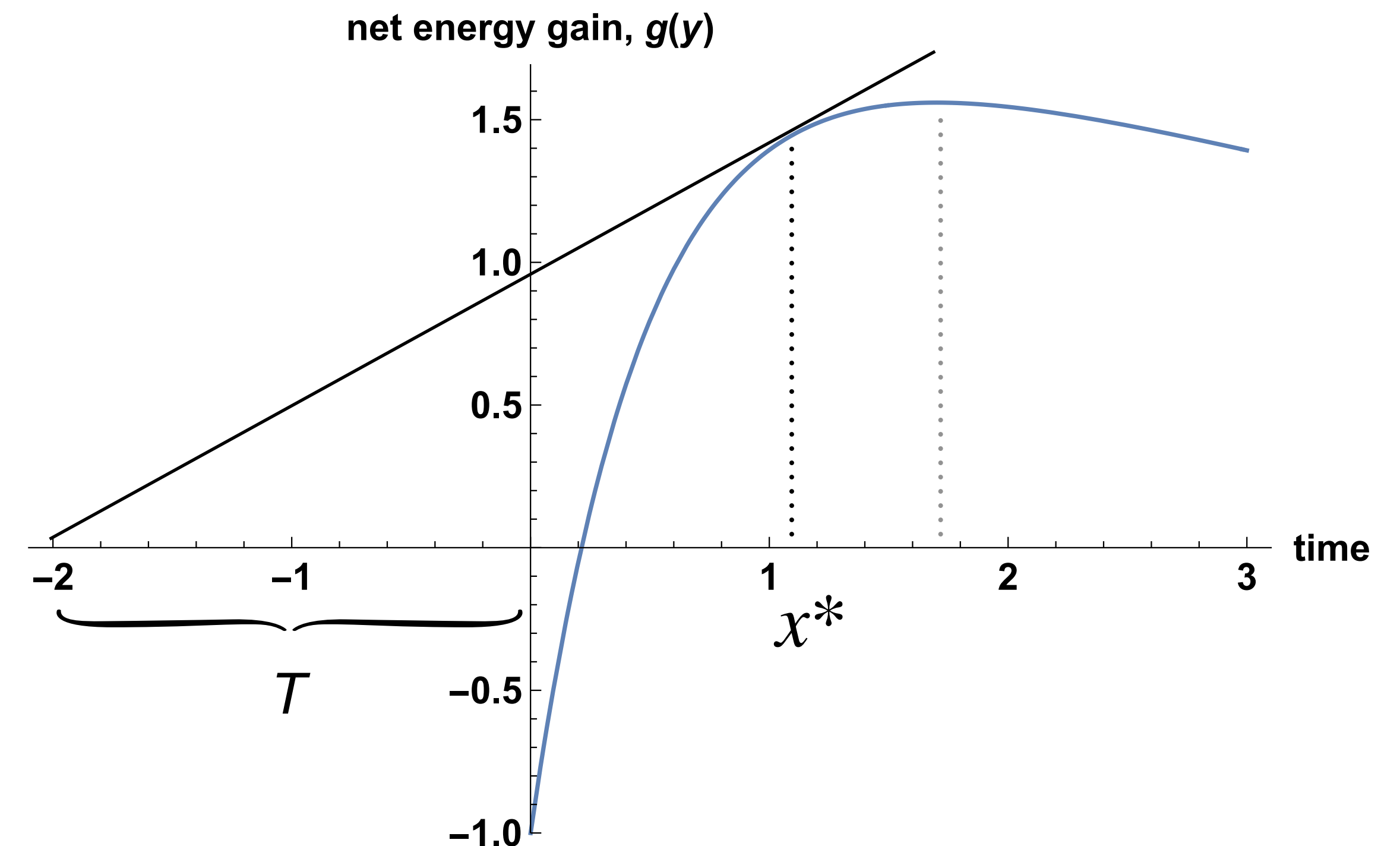
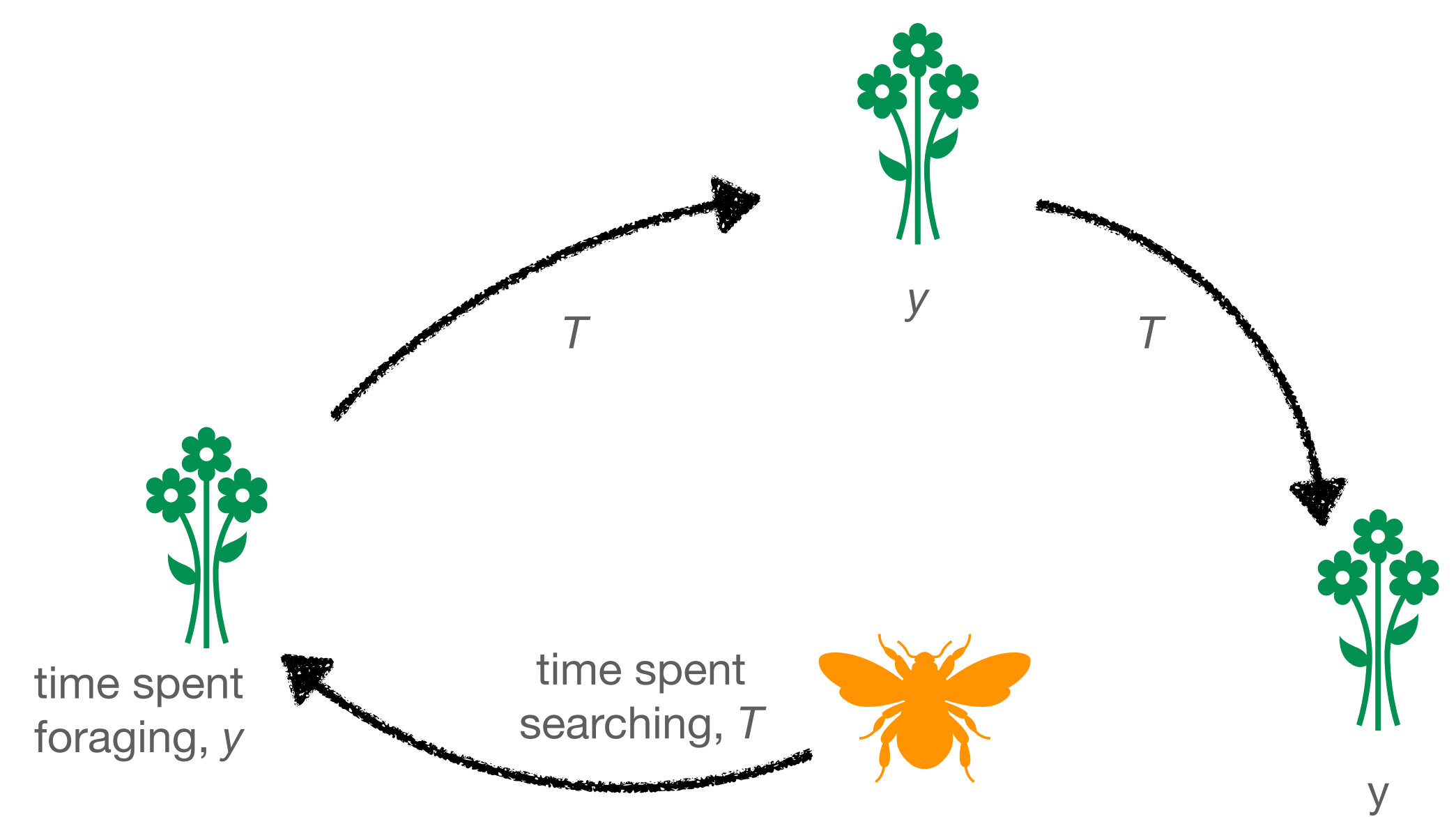
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An animal should leave when the marginal
(or instantaneous) rate of energy gain
 $g'(x^*)$ has fallen to the rate of energy gain
 $R(x^*)$

When selection favours risky foraging?

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Variation in relationship with uncertainty



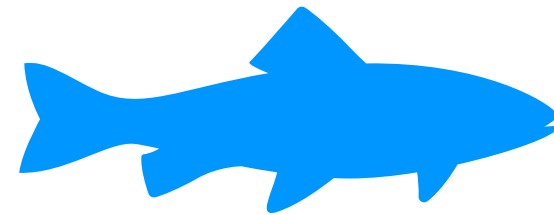
When selection favours risky foraging?

State-dependent payoffs and uncertainty

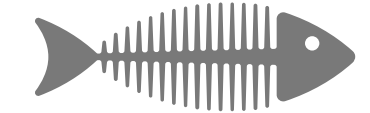
When selection favours risky foraging?

State-dependent payoffs and uncertainty

High condition
e.g., well-fed



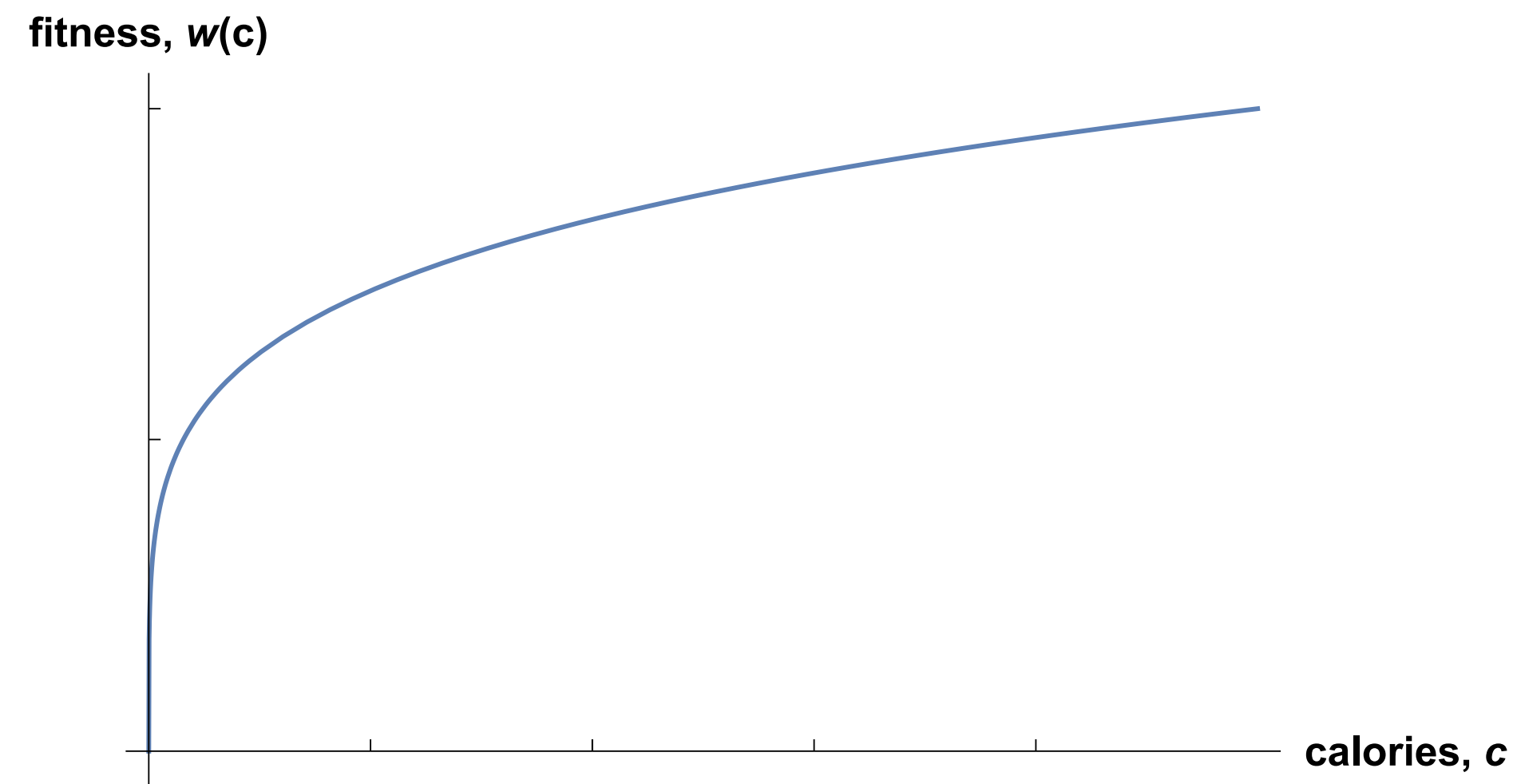
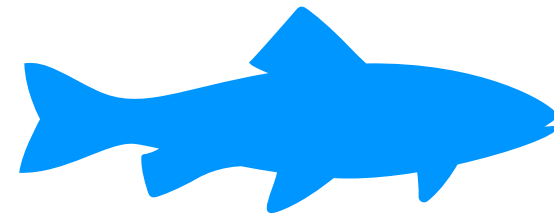
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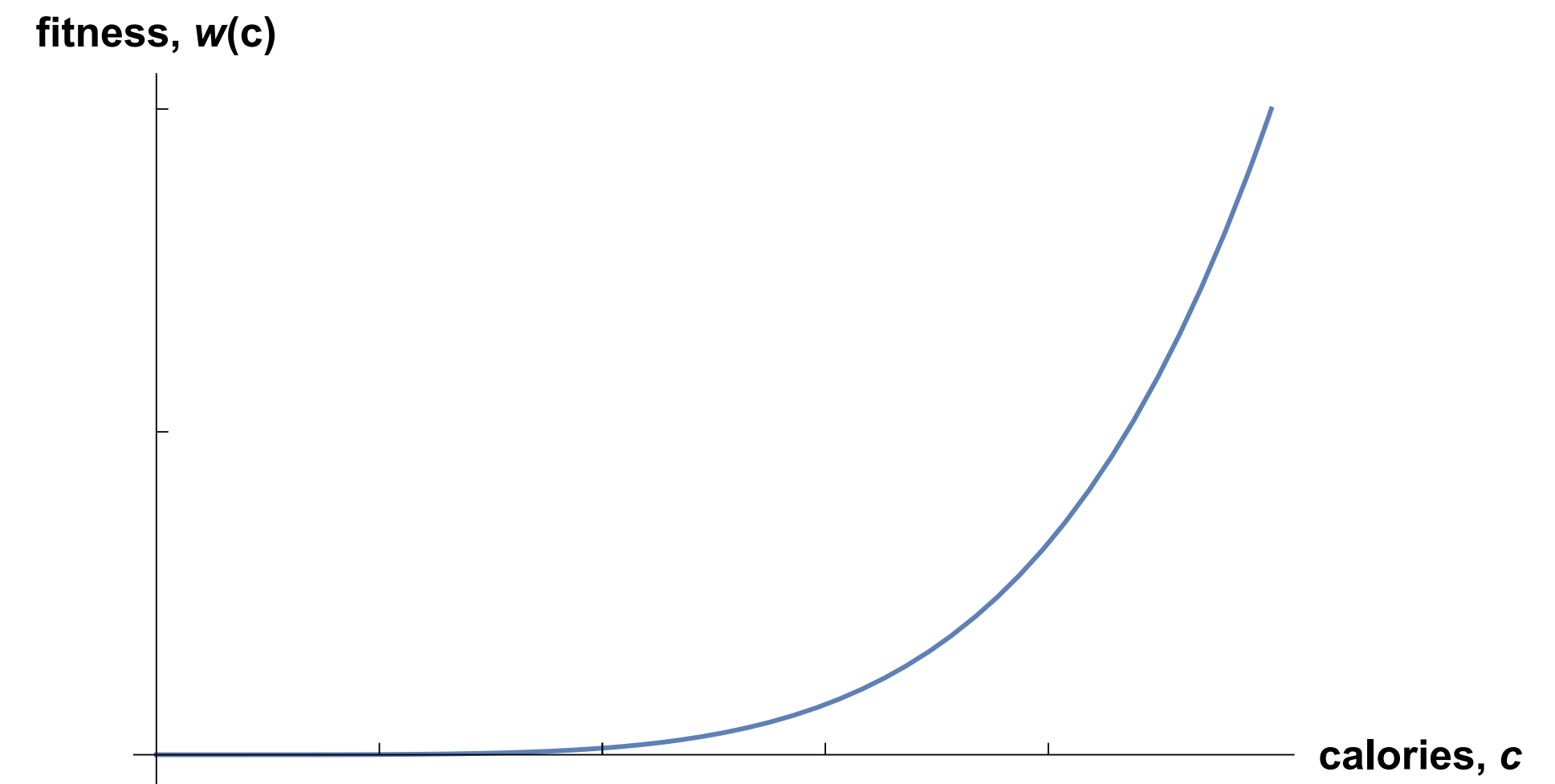
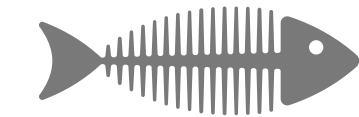
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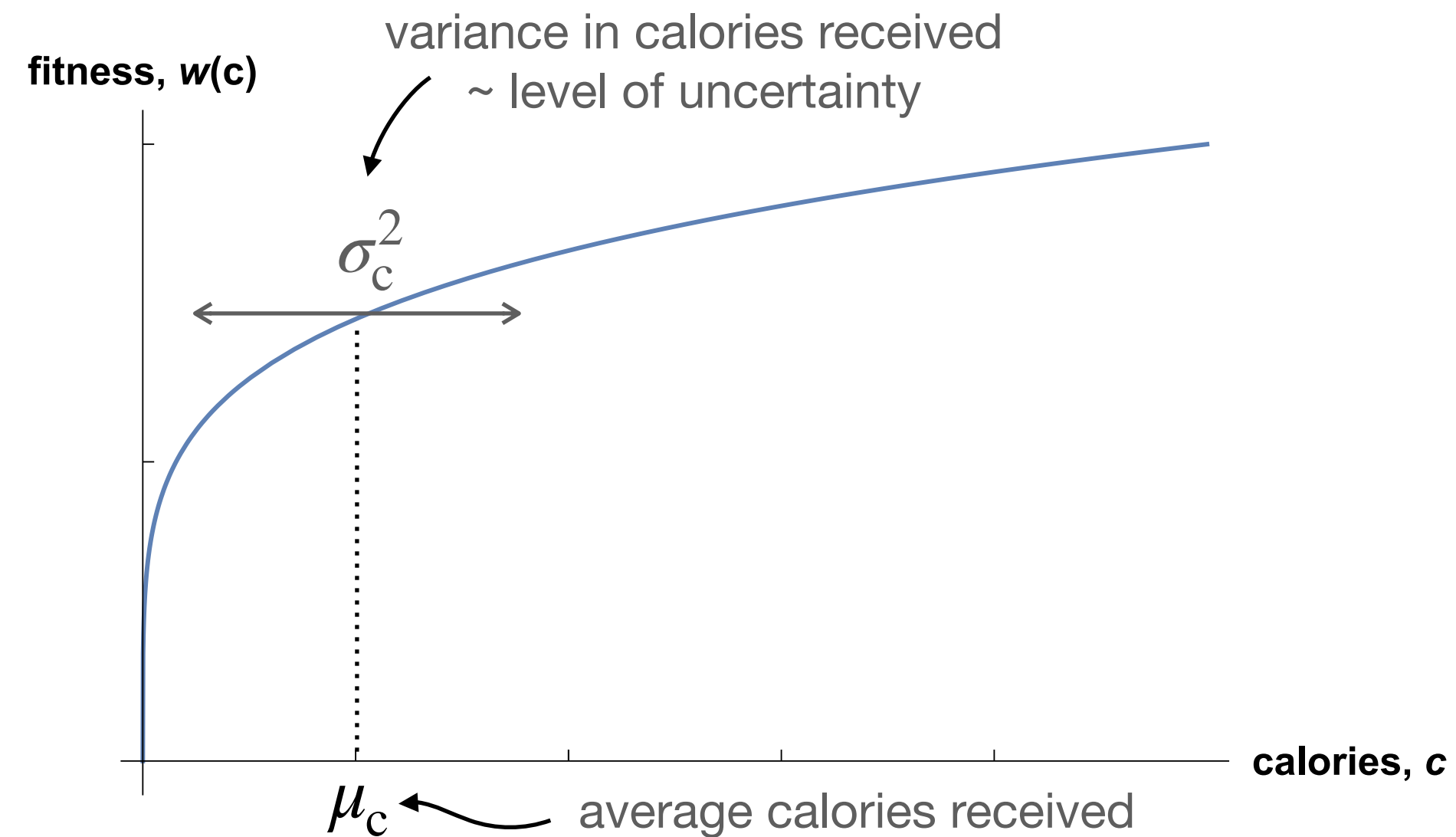
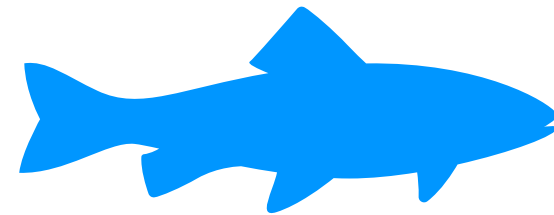
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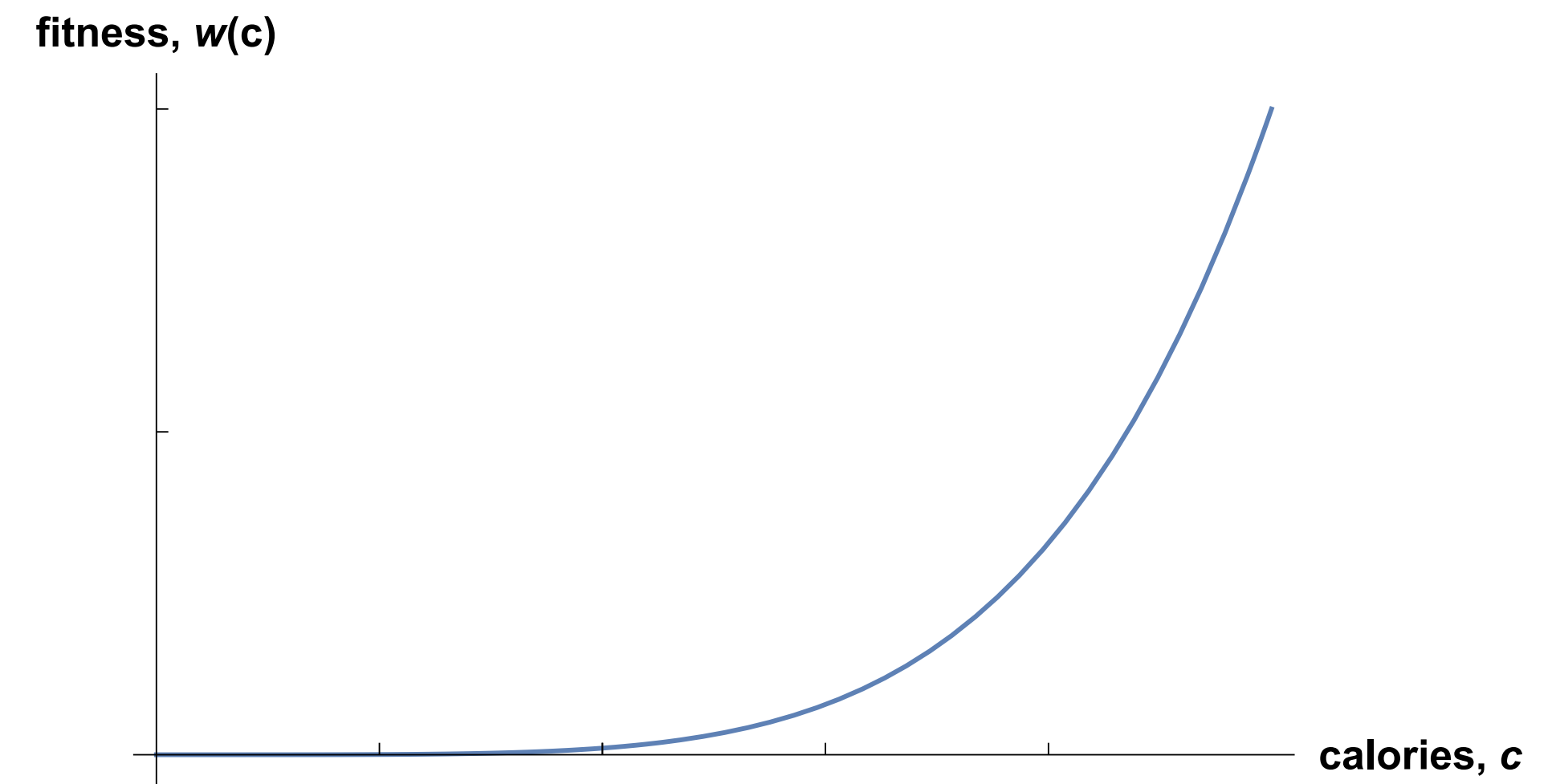
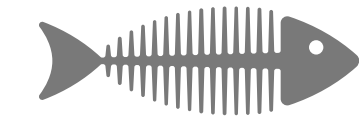
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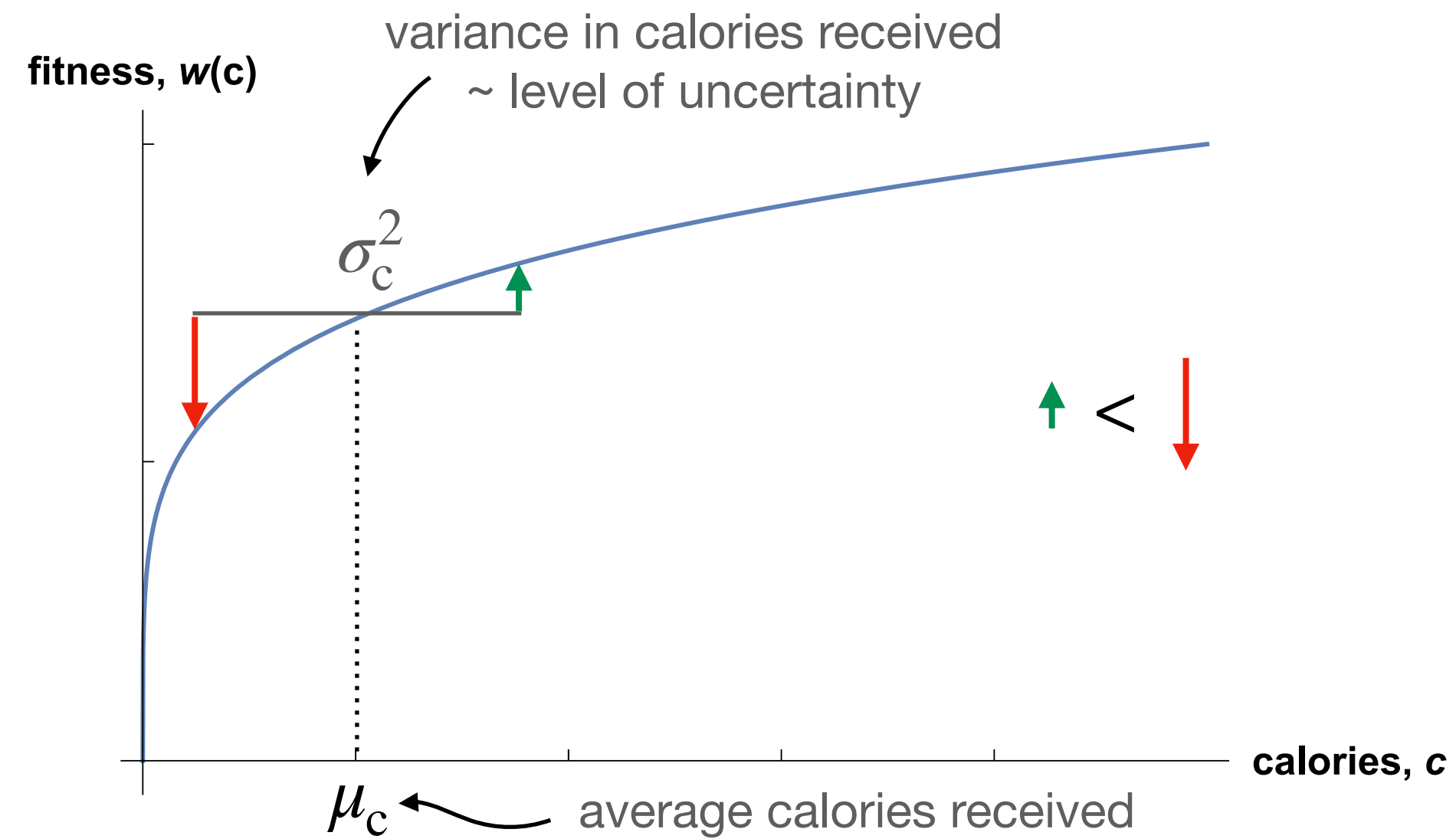
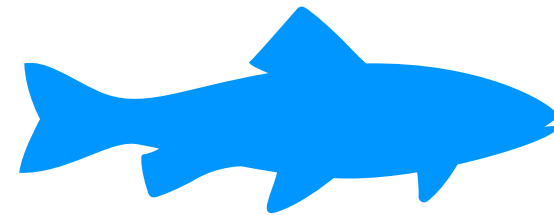
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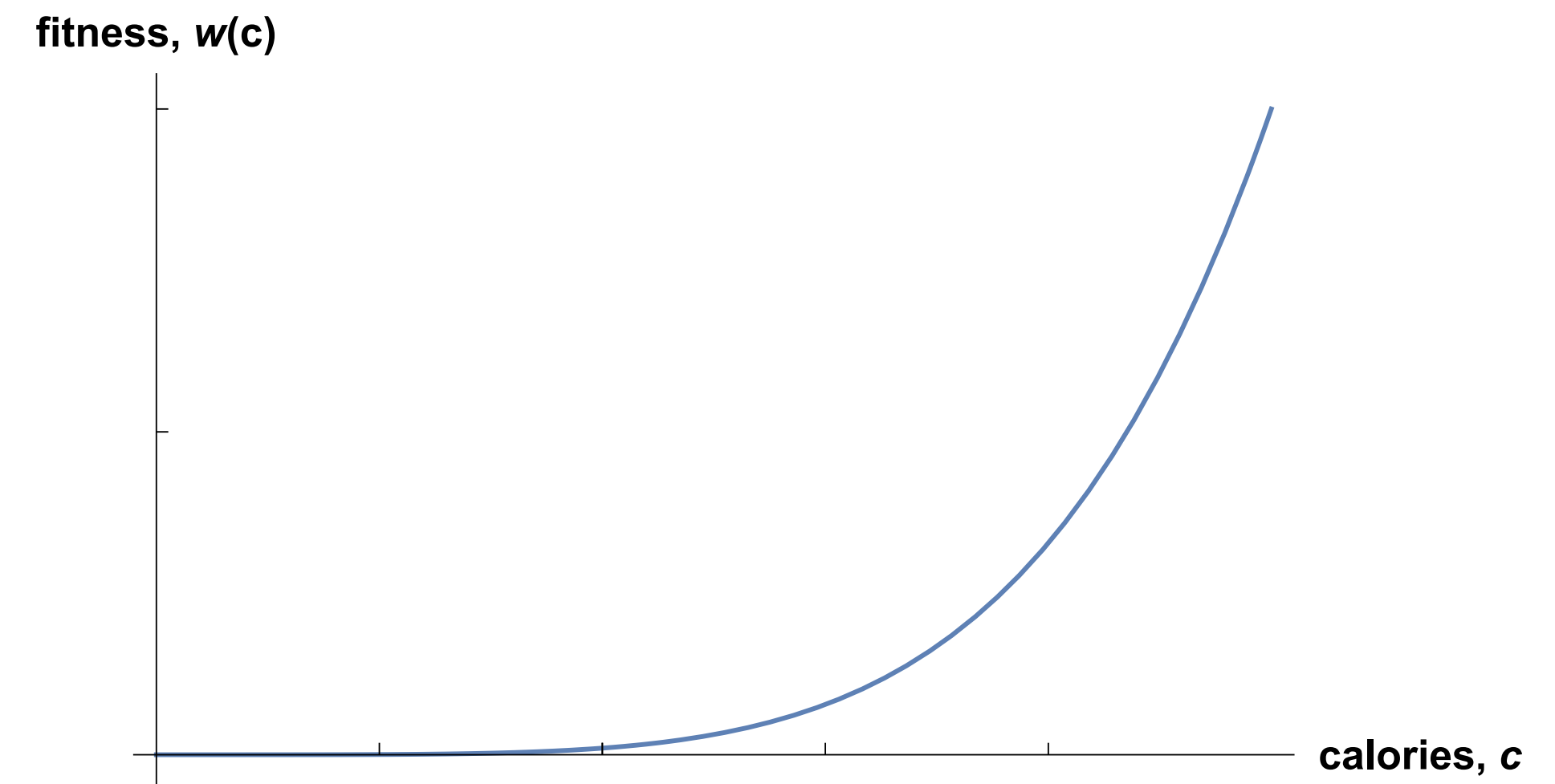
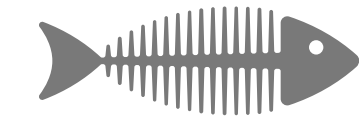
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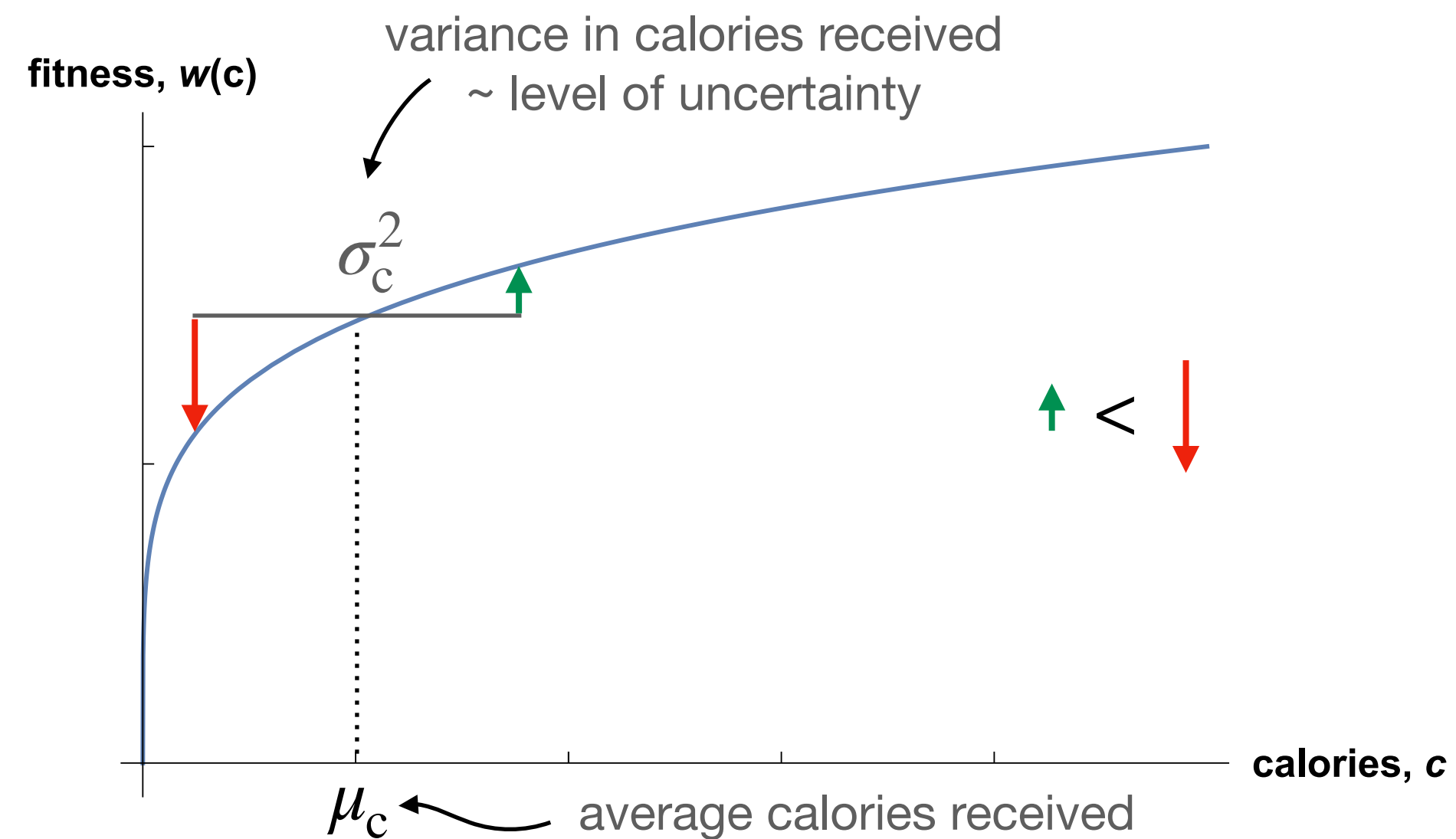
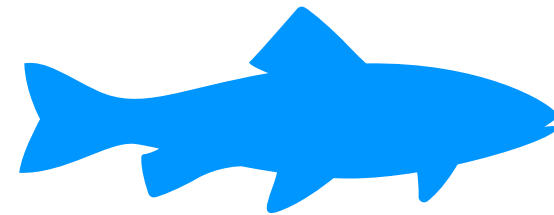
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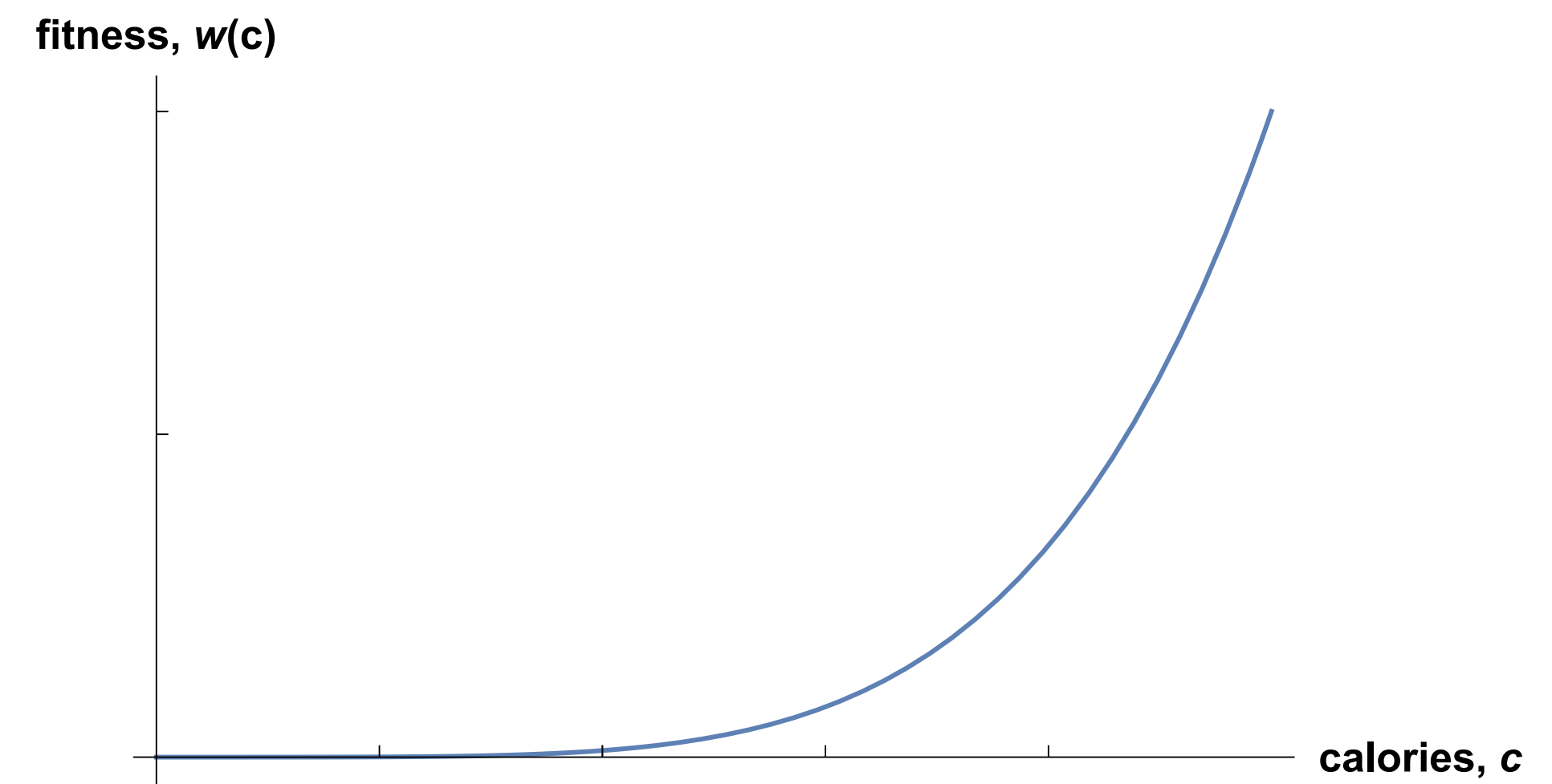
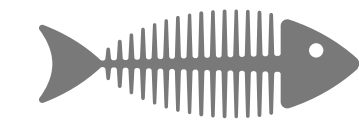
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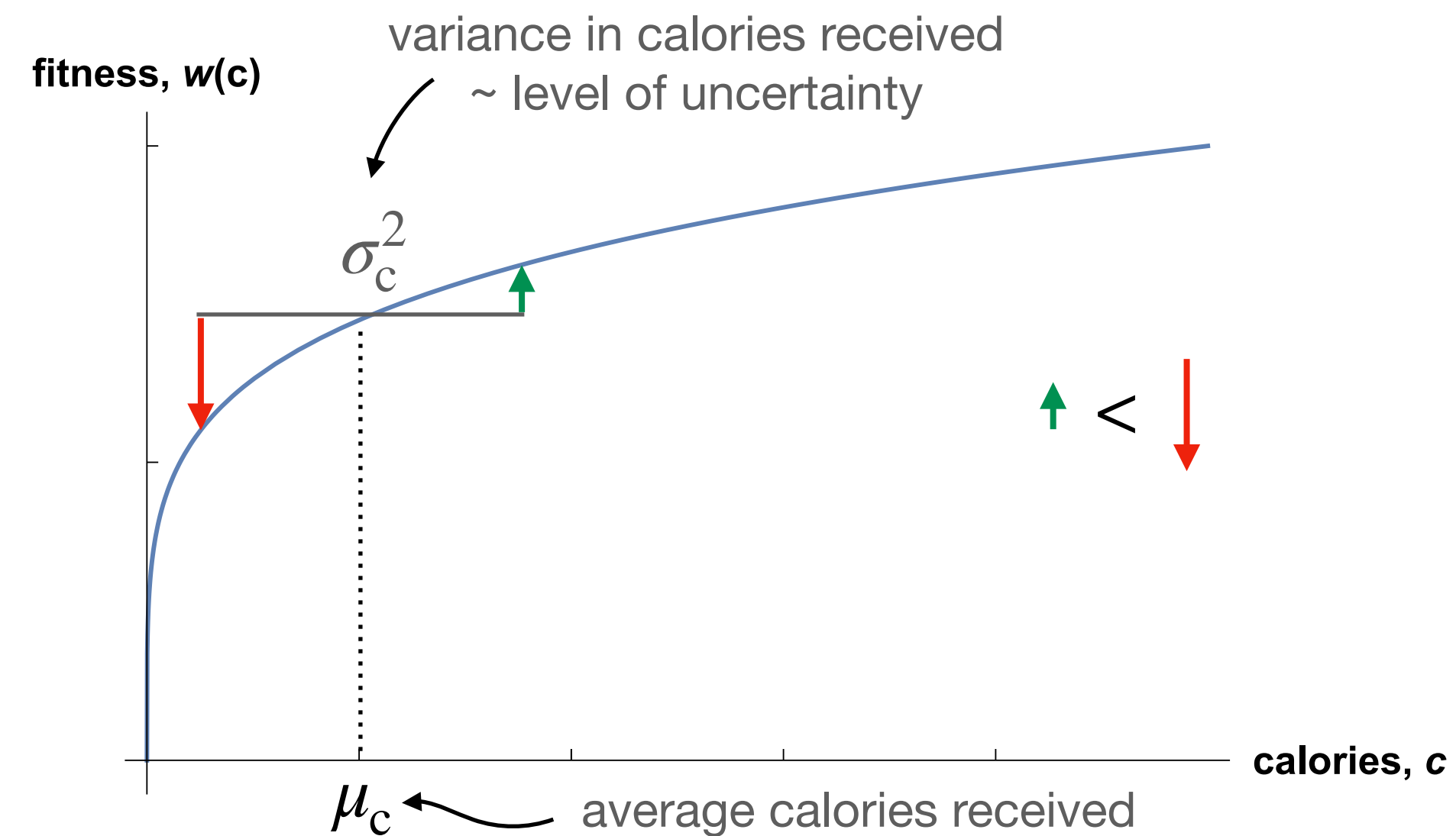
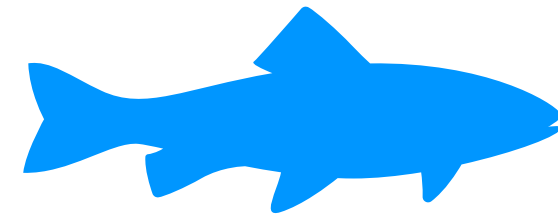


Risk not worth taking: fitness cost of bad times outweighs fitness benefits of good times

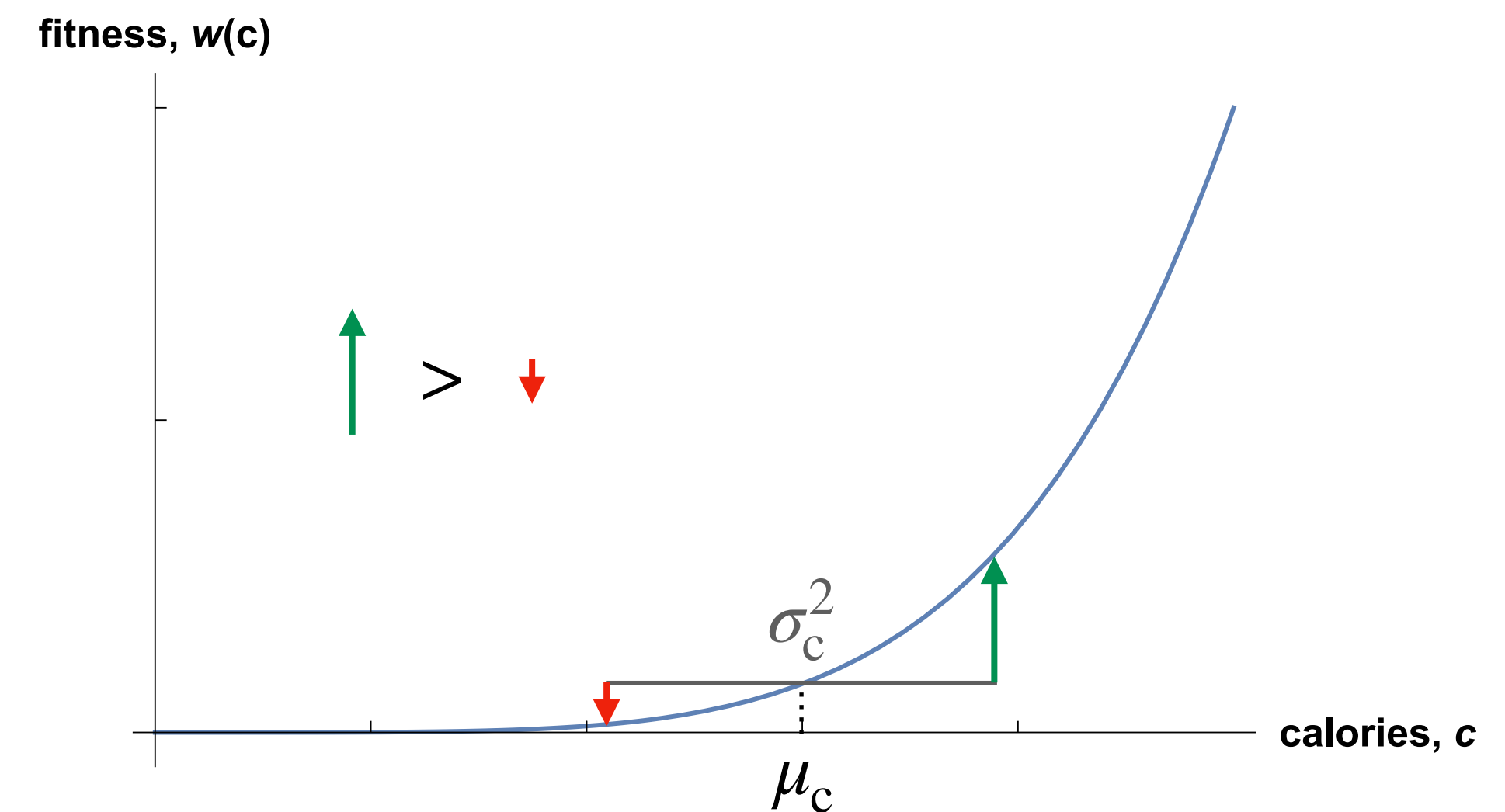
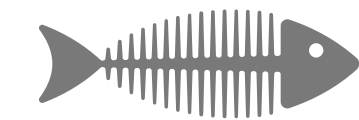
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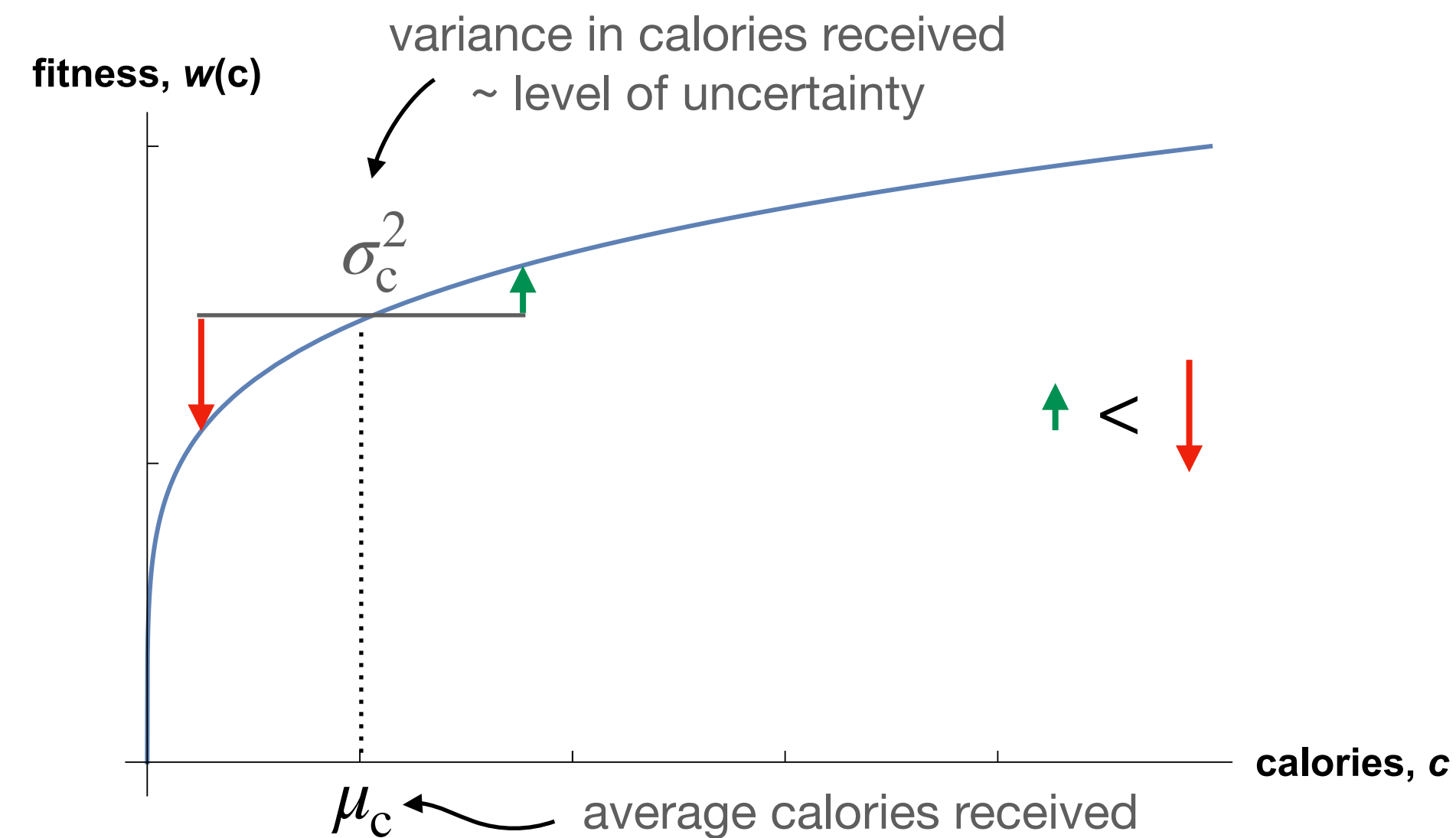
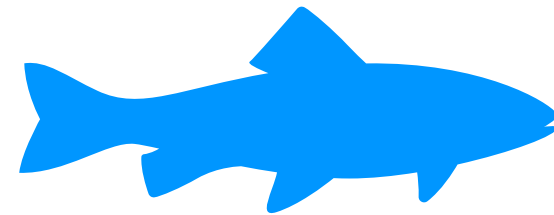


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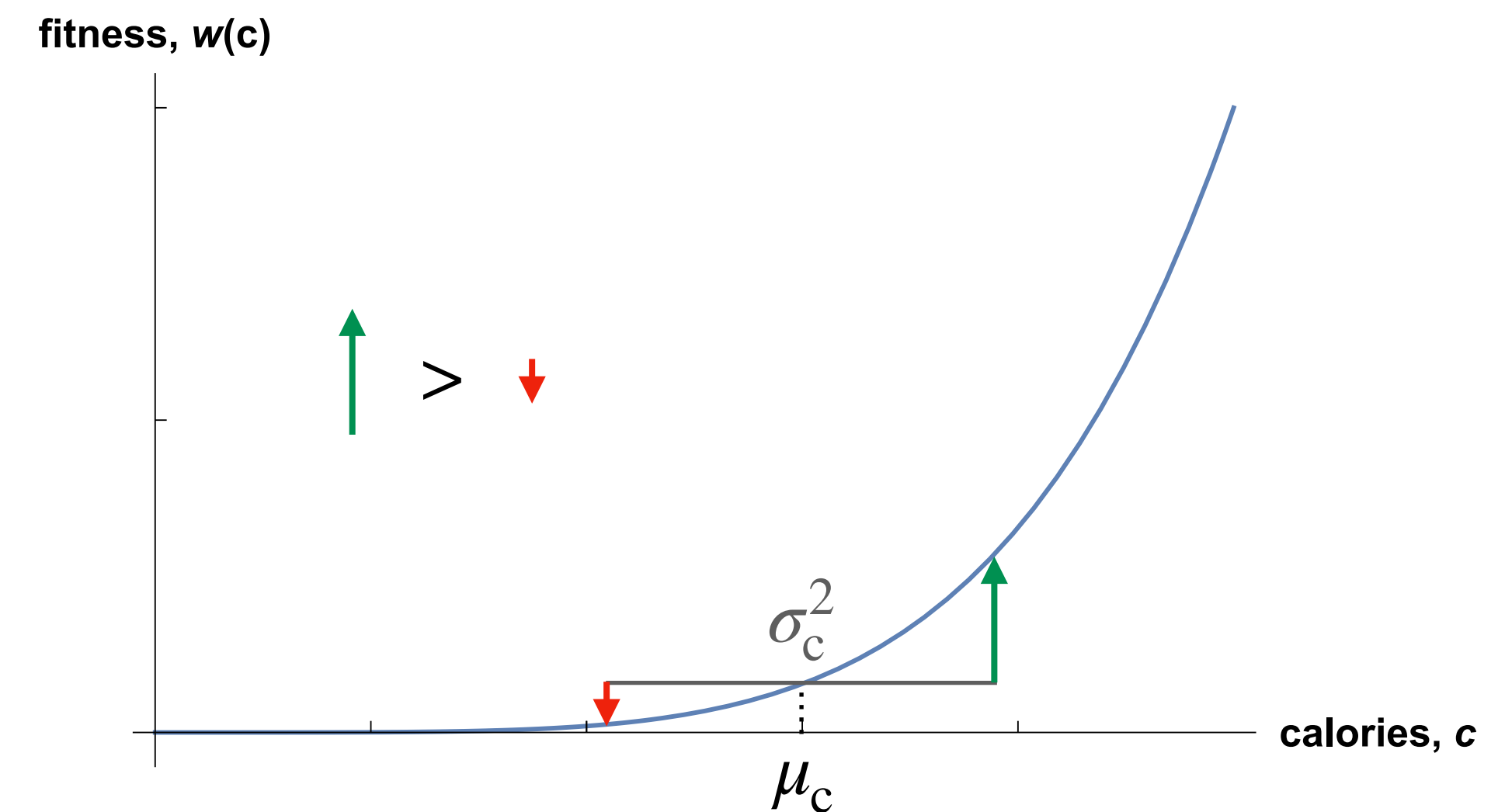
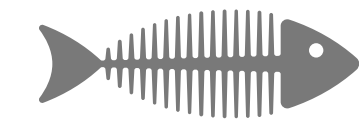
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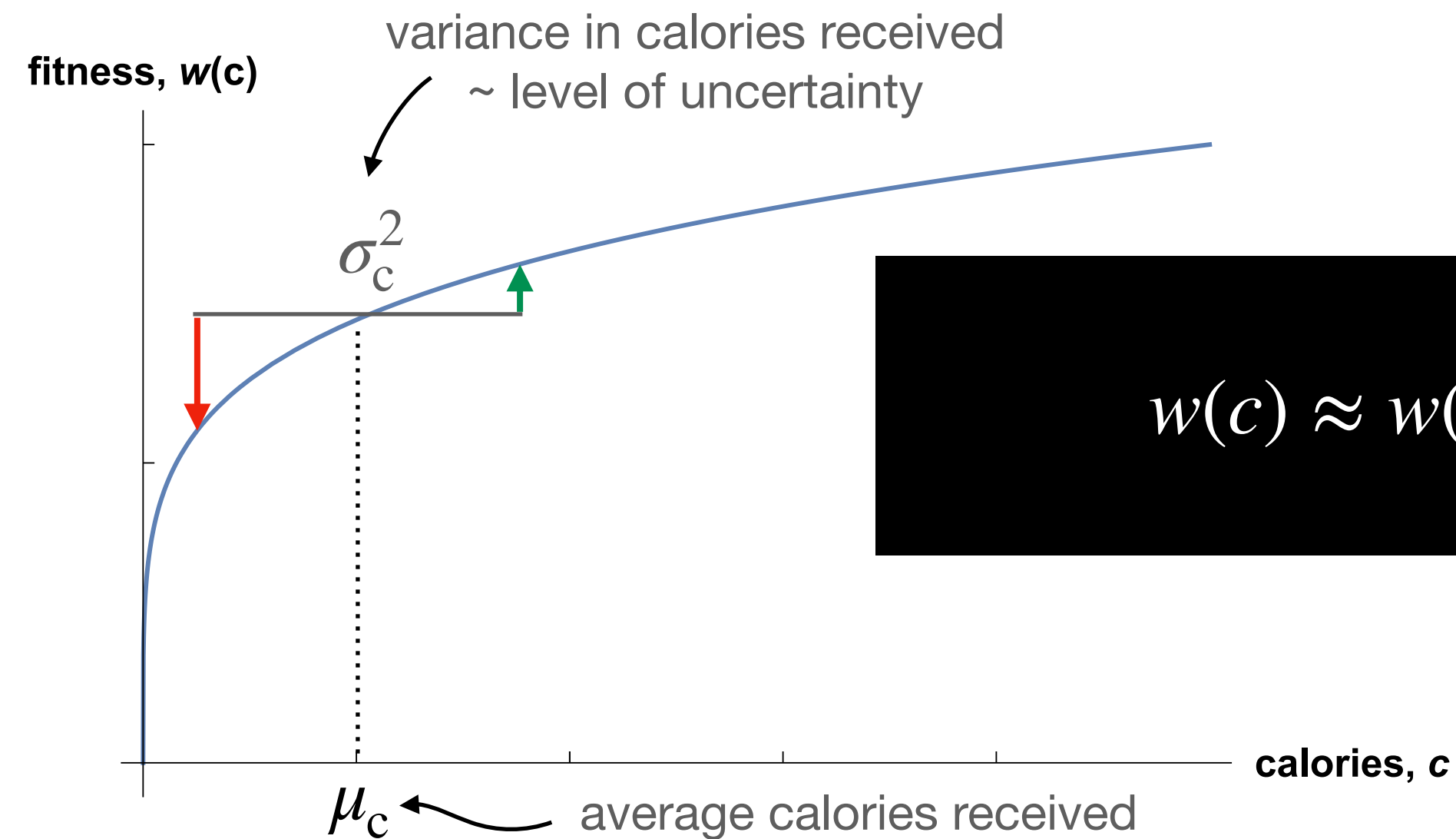
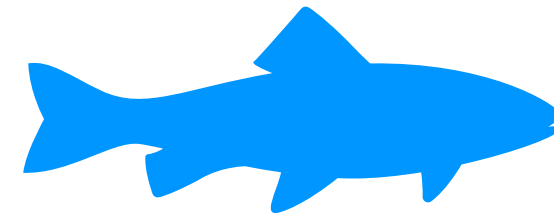
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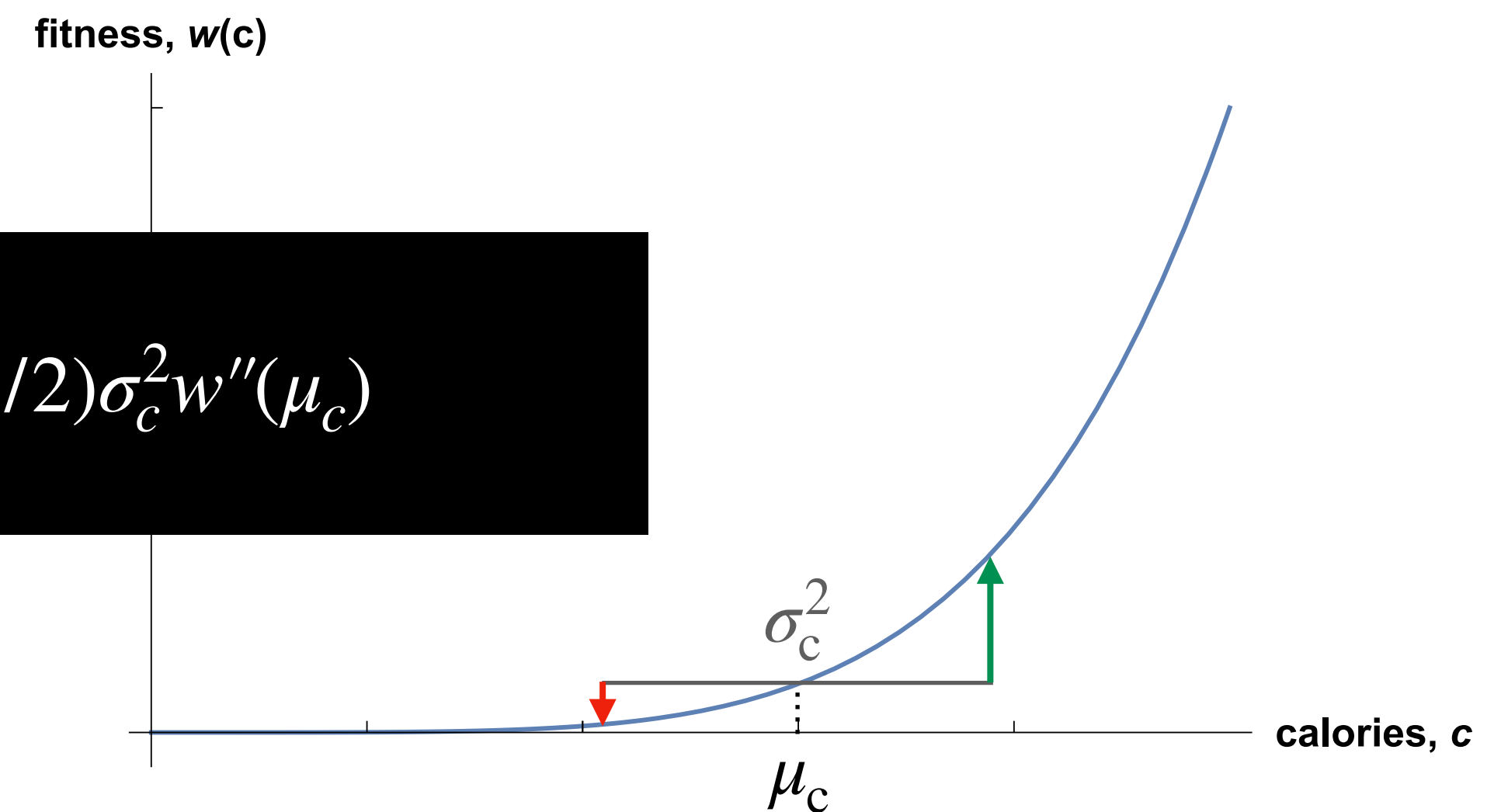
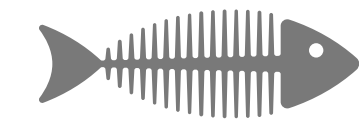
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$$w(c) \approx w(\mu_c) + (1/2)\sigma_c^2 w''(\mu_c)$$

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The exploitation of renewable resources

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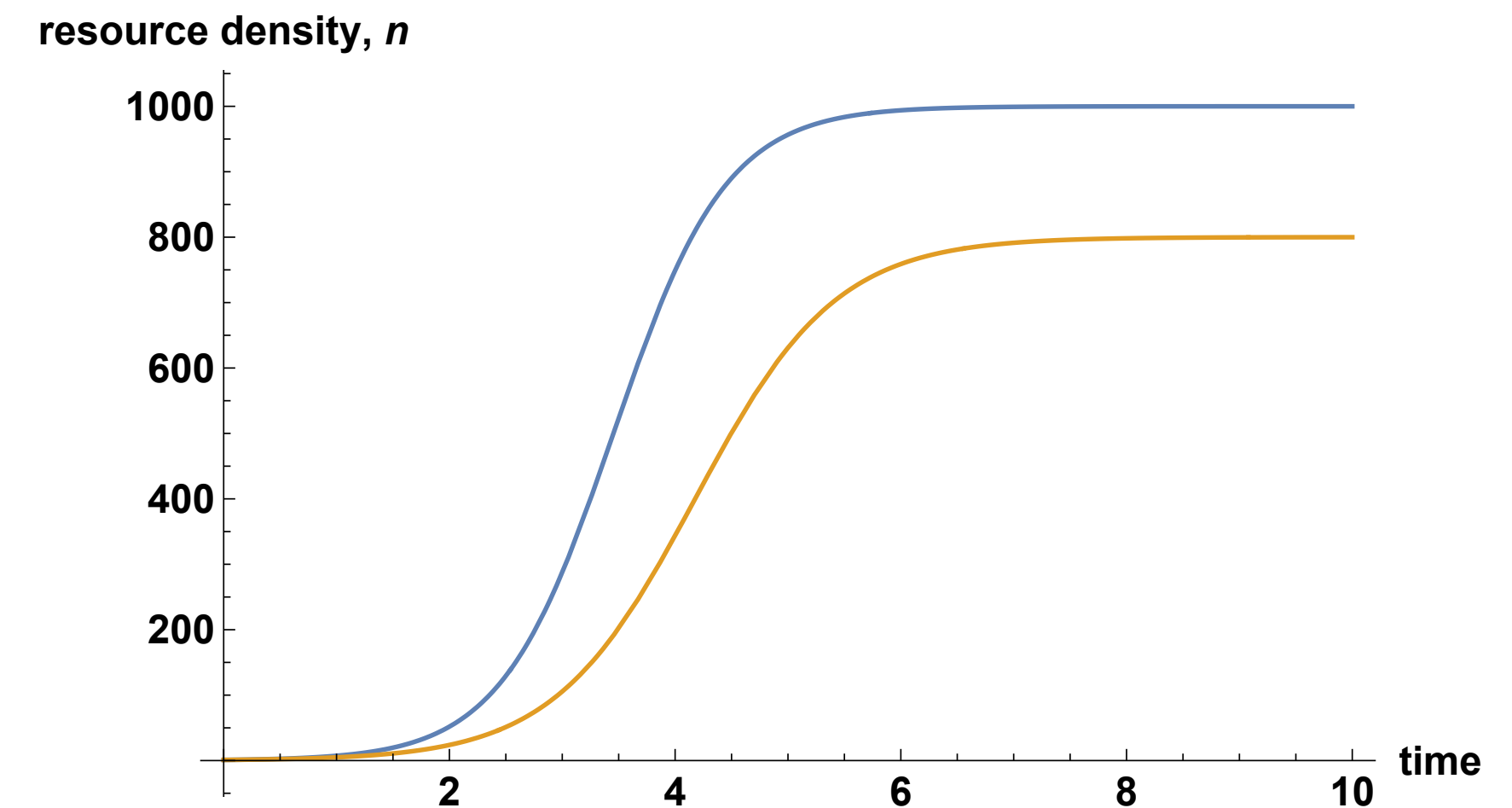
Schaefer's model

- Biotic resource with density n ,

$$\frac{dn}{dt} = \underbrace{r \left(1 - \frac{n}{K} \right) n}_{\text{logistic growth}} - \underbrace{n_c h(x) n}_{\text{foraging function}}$$

logistic growth

harvesting by population
of n_c consumers with
foraging effort x

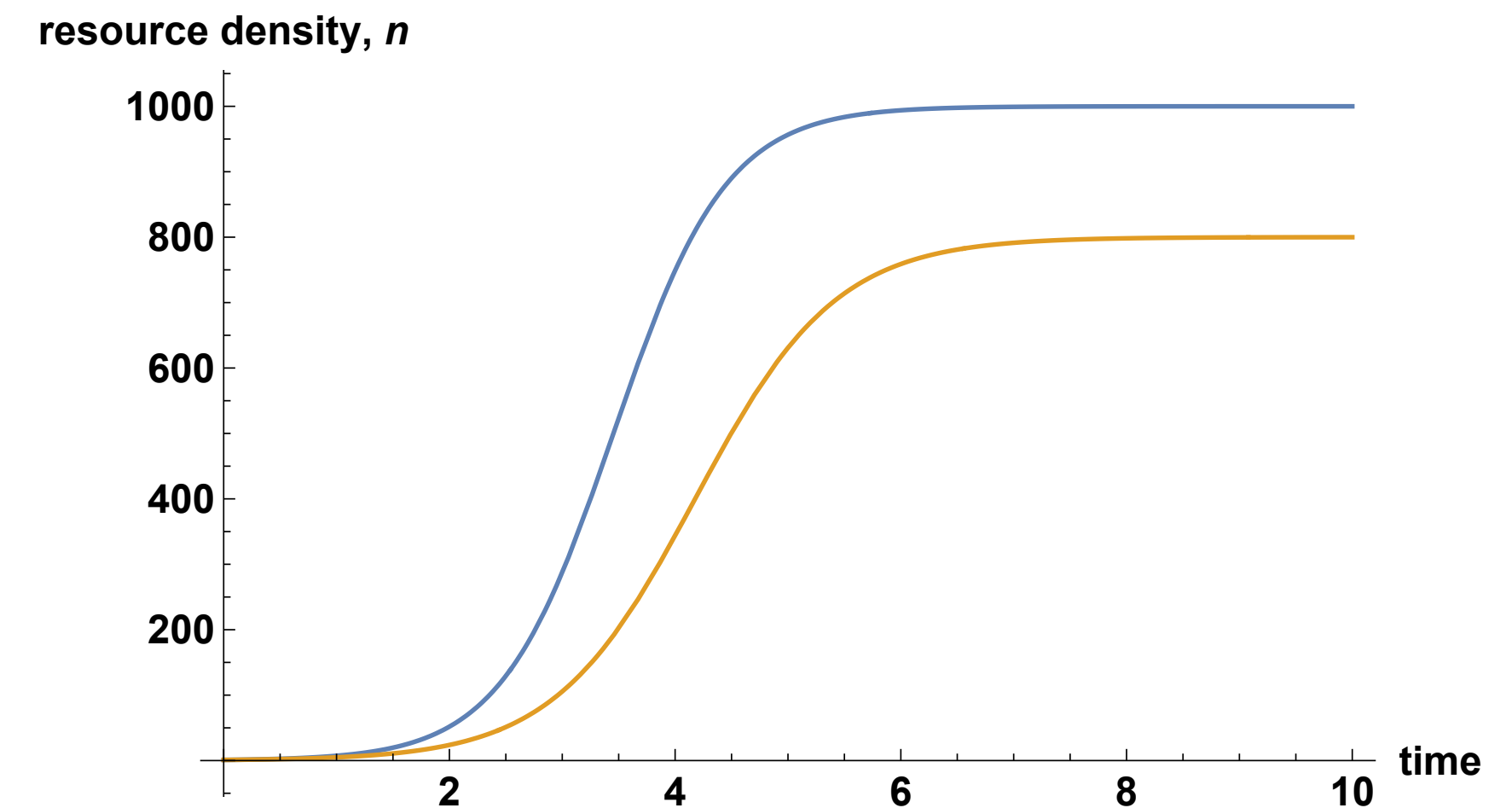


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- Equilibrium resource density $\hat{n}(x)$ such that

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The exploitation of renewable resources

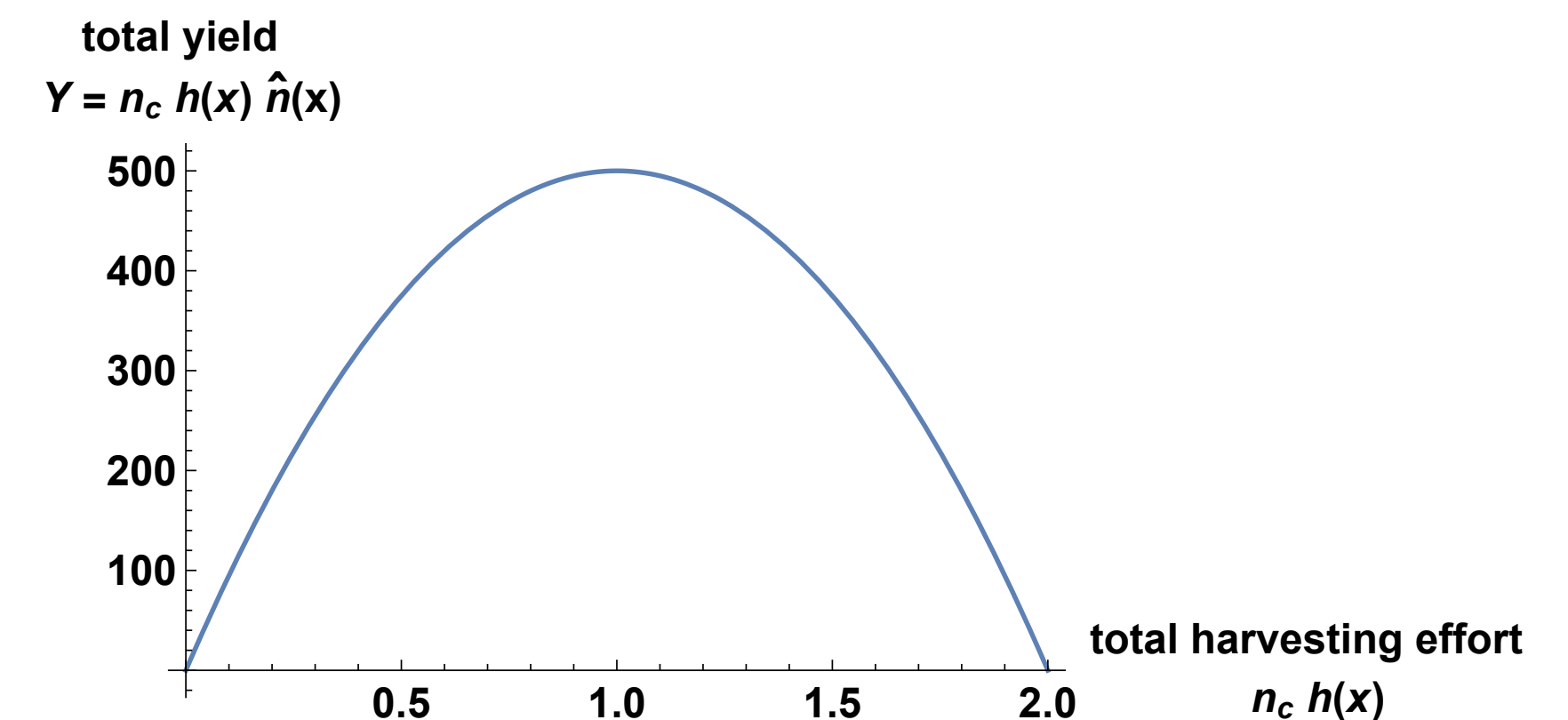
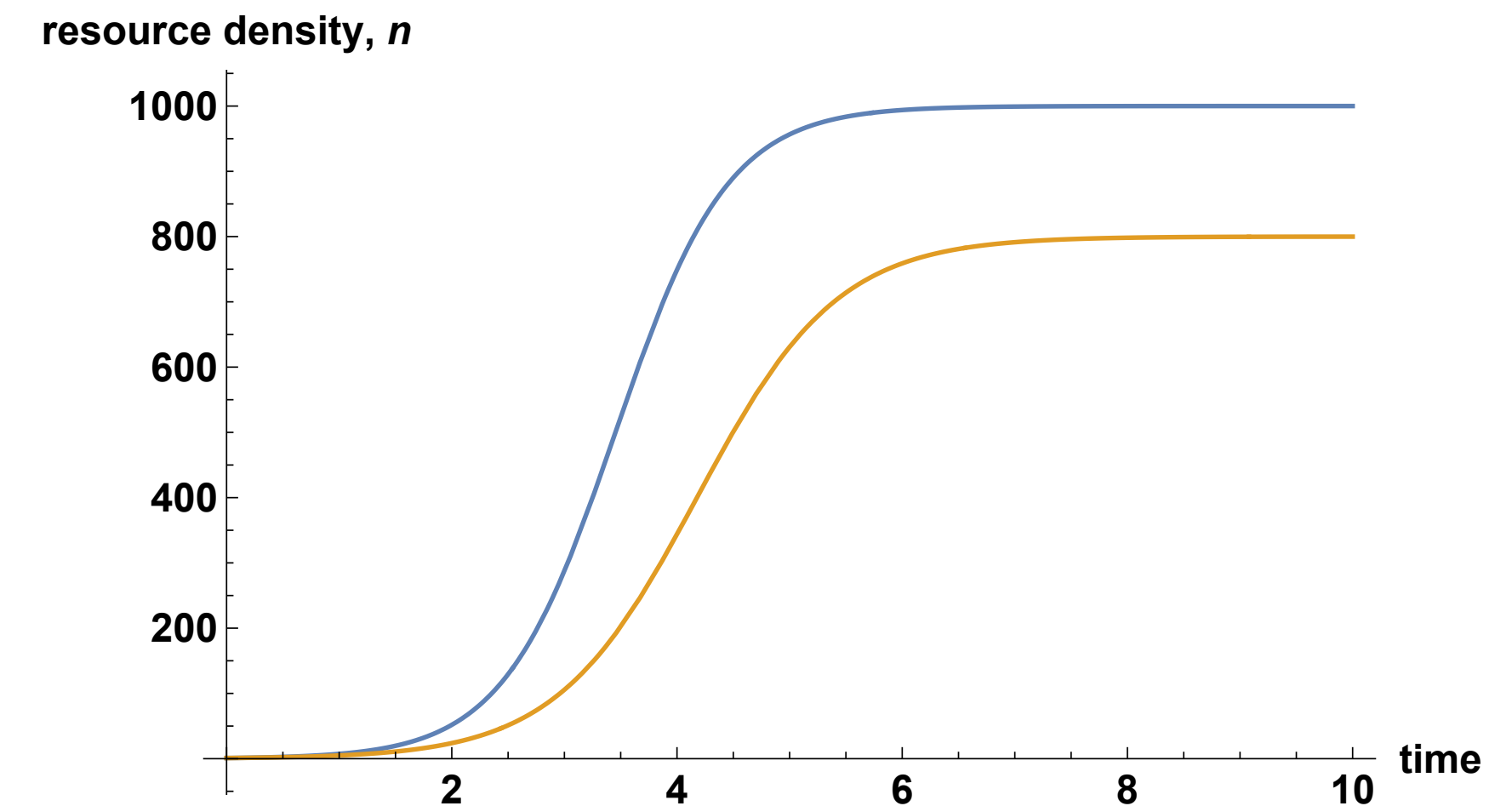
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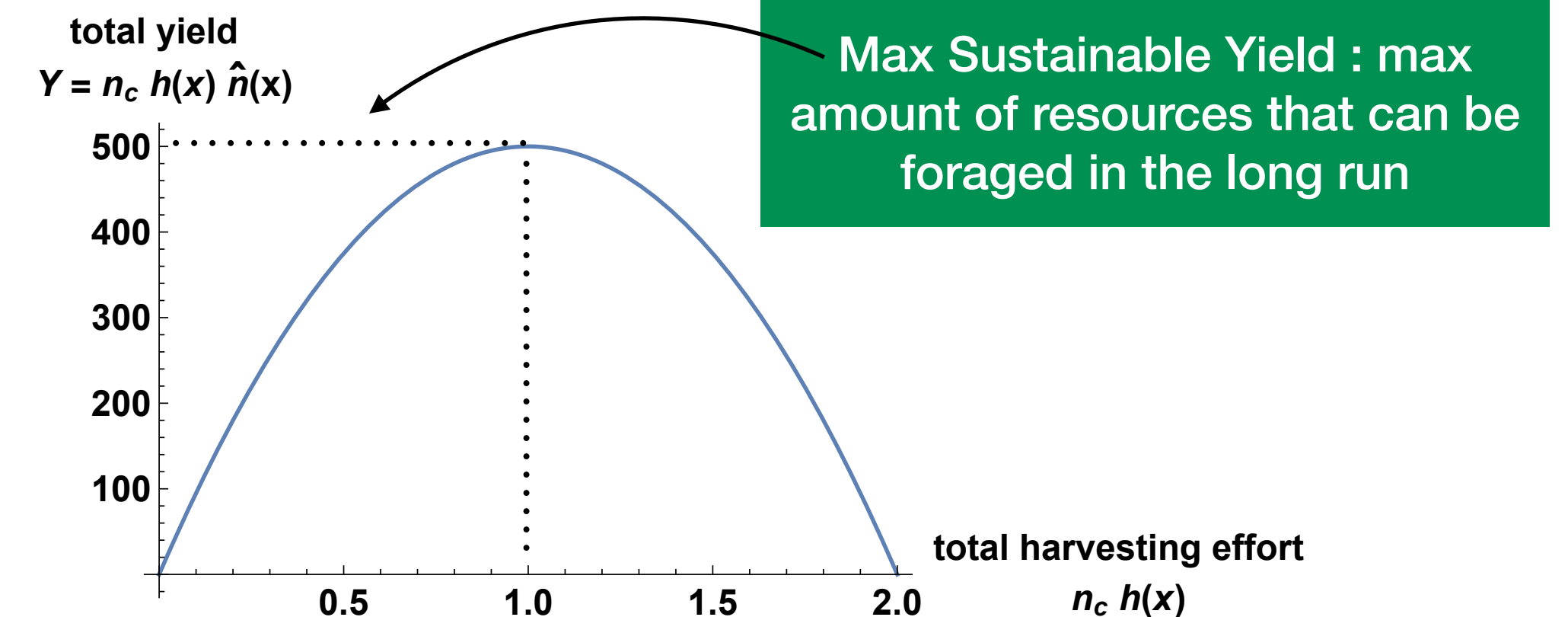
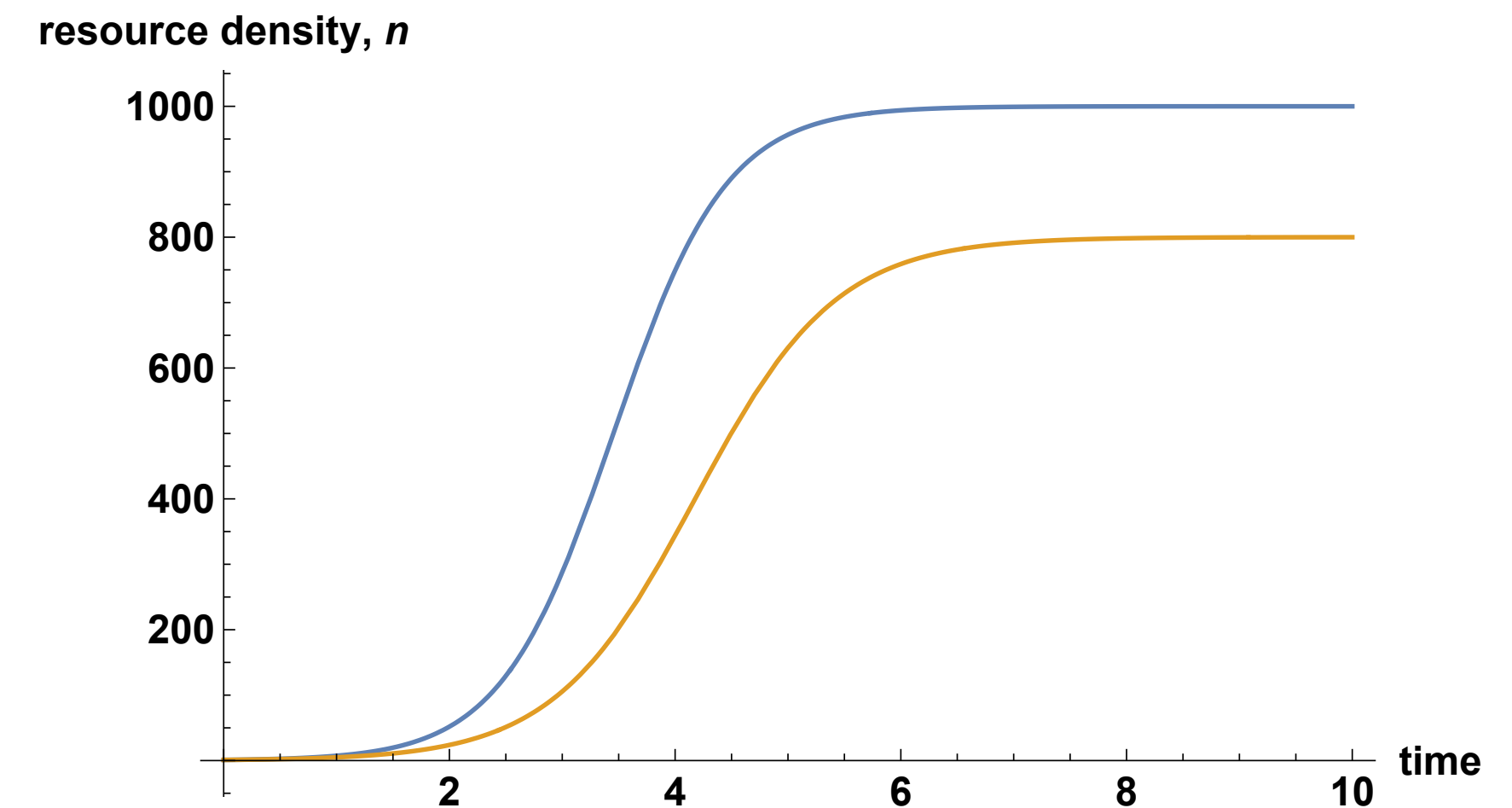
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foraging function

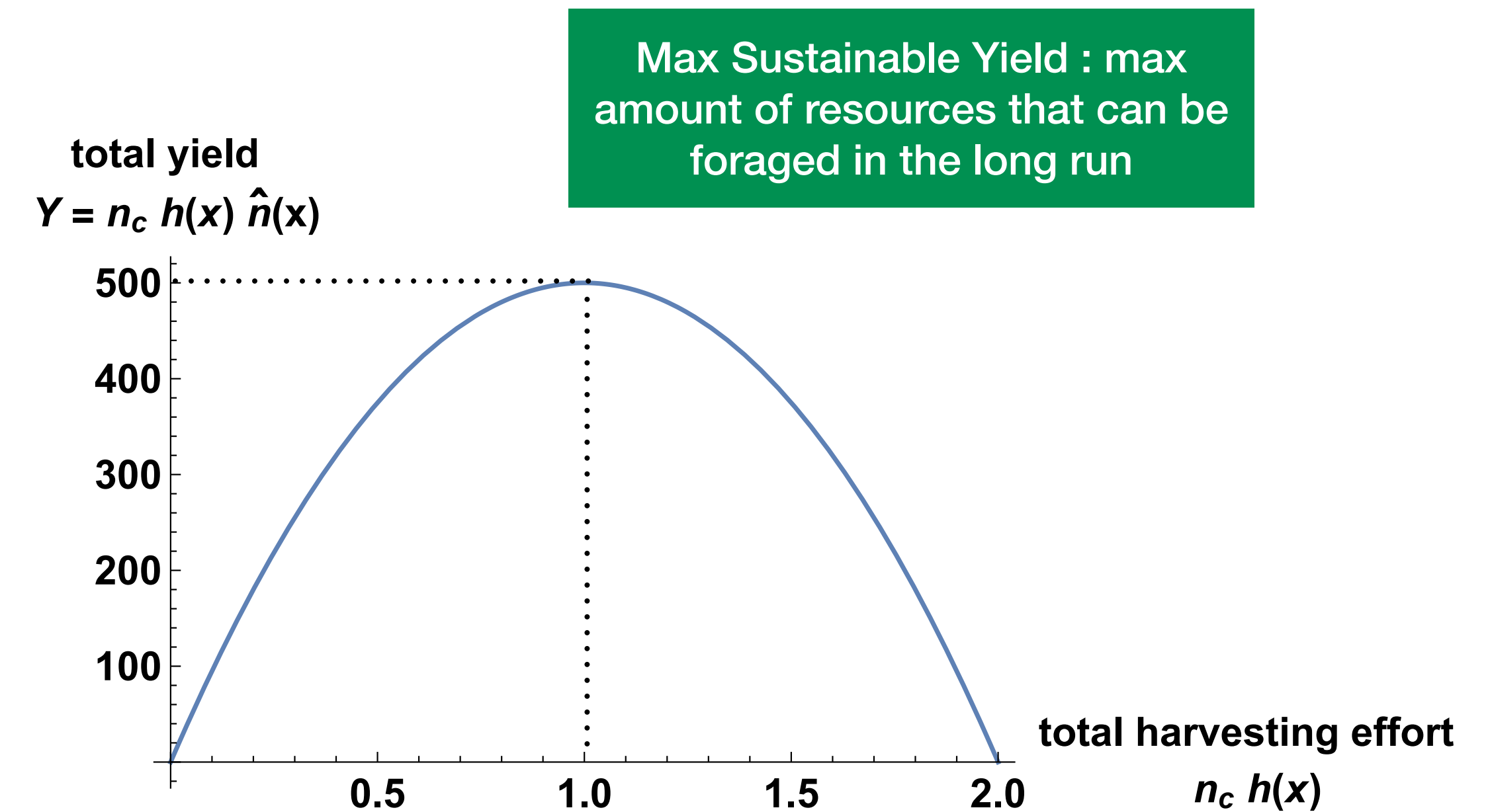
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The exploitation of renewable resources

MSY and over-consumption

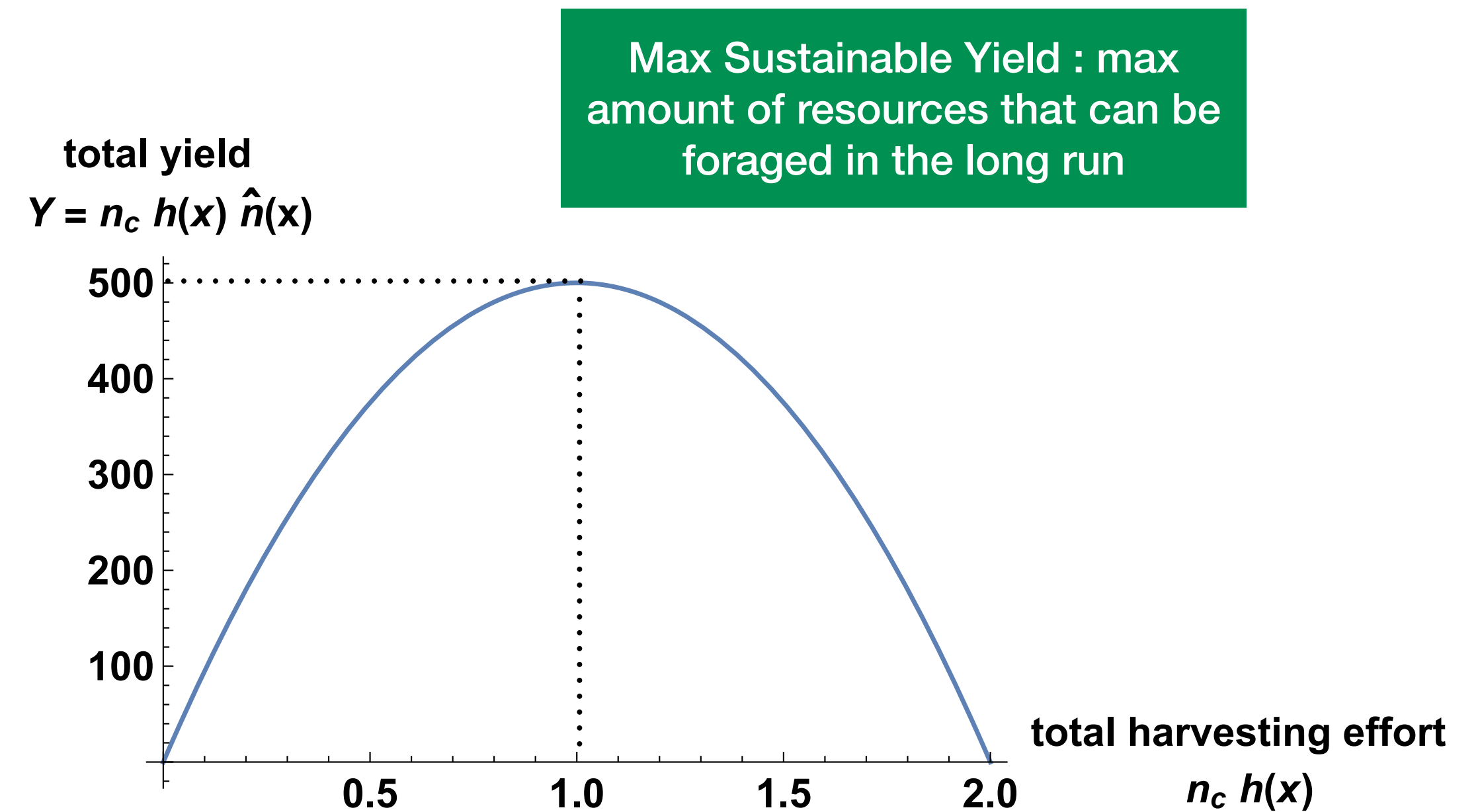


The exploitation of renewable resources

MSY and over-consumption

• Total yield = $n_c h(x) \times \hat{n}(x) = n_c x \times K \left(1 - n_c \frac{x}{r} \right)$

h(x) = x



The exploitation of renewable resources

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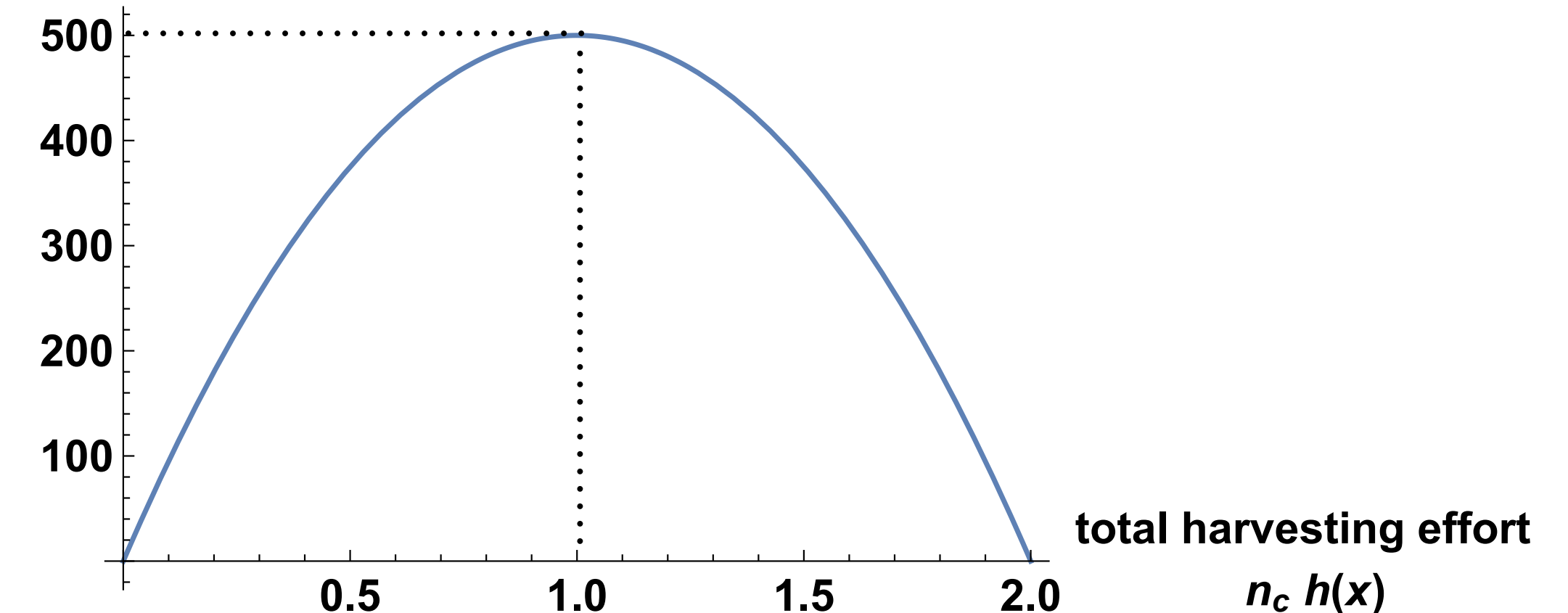
- x_{MSY} : Foraging effort that maximises total yield =

$$x_{\text{MSY}} = \frac{1}{n_c} \frac{r}{2}$$

- $\text{MSY} = n_c h(x_{\text{MSY}}) \times \hat{n}(x_{\text{MSY}}) = \frac{Kr}{4}$

- Resource density = $\hat{n}(x_{\text{MSY}}) = \frac{K}{2}$

total yield
 $Y = n_c h(x) \hat{n}(x)$

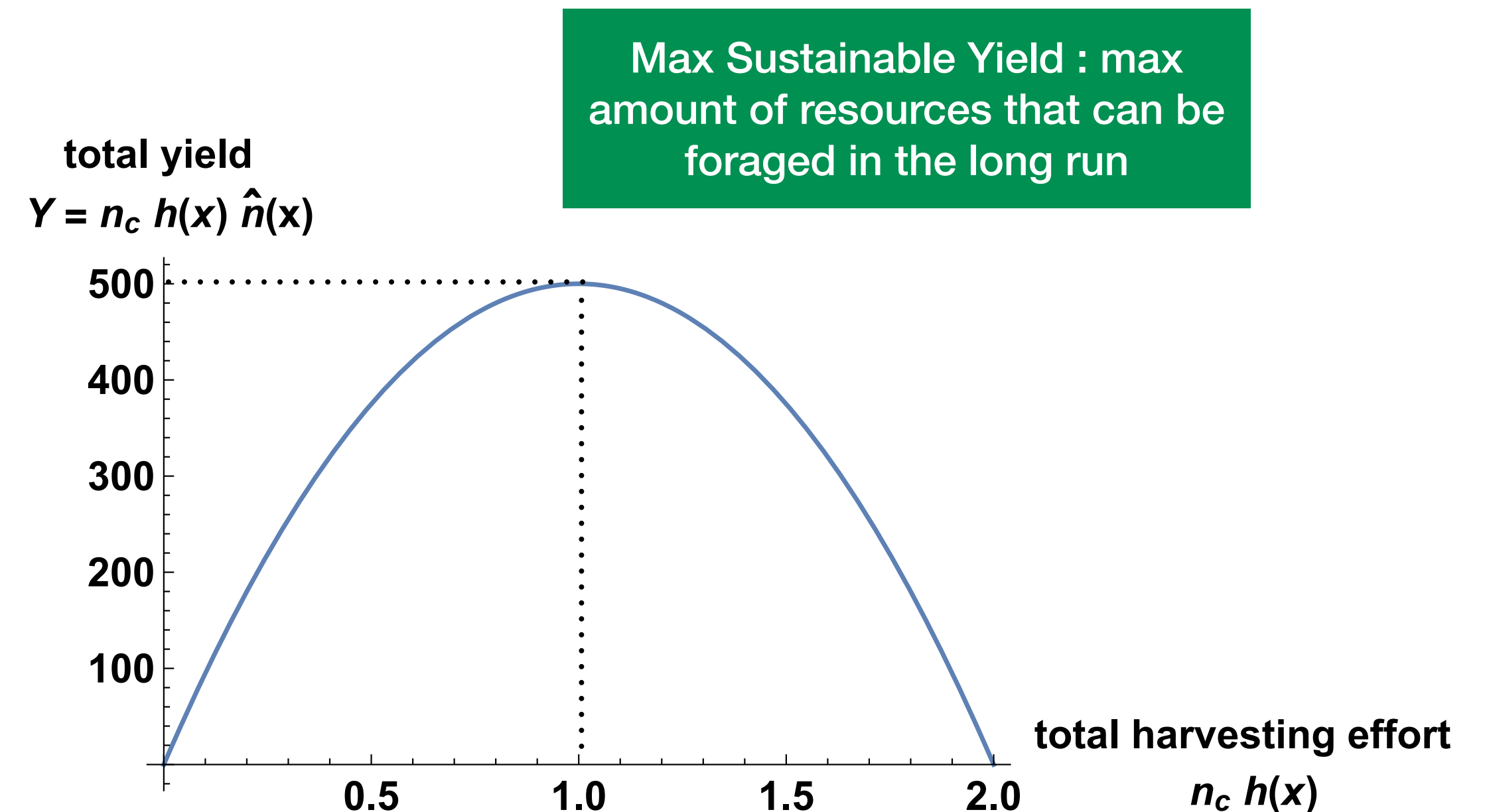


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- $$\text{Resource density} = \hat{n}(x_{\text{MSY}}) = \frac{K}{2}$$
- Any effort above x_{MSY} amounts to over-exploitation.



The exploitation of renewable resources

How evolution shapes foraging of biotic resources

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- Well-mixed population where individuals all exploit the same resource and compete with one another.

The exploitation of renewable resources

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- Fitness of a mutant with foraging effort y in a resident population x ,

individual yield - individual cost of effort

$$w(y, x) \propto y\hat{n}(x) - c(y)$$

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$$\bullet \quad x^* = x_{\text{MSY}} \frac{2Kn_c}{Kn_c + c_0r} \quad \begin{array}{l} h(x) = x \\ c(x) = \frac{c_0}{2}x^2 \end{array}$$

The exploitation of renewable resources

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$$\begin{aligned} h(x) &= x \\ c(x) &= \frac{c_0}{2} x^2 \end{aligned}$$

When cost is large, $c_0 \geq \frac{Kn_c}{r}$ then $x^* \leq x_{\text{MSY}}$.

Otherwise, $x^* > x_{\text{MSY}}$.

When $c_0 = 0$, evolution leads to resource extinction.

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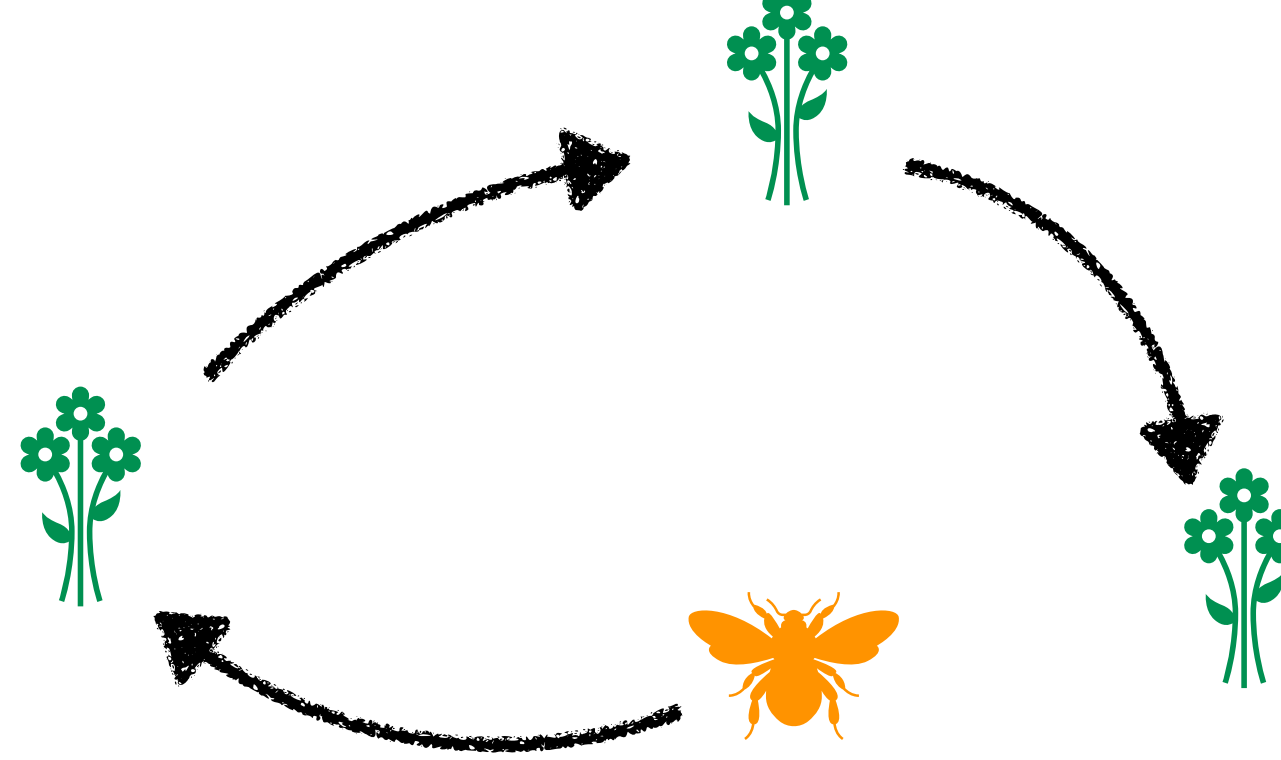
Due to competition, evolution typically leads to over-exploitation and lower yield than if individuals were coordinated.

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Summary



- Marginal value theorem allows to understand when an organism should leave for new pastures: leave when the *marginal* rate of energy gain has fallen to the total rate of gain.
- Risky foraging behaviours can be explained from state dependent payoffs where the fitness of low condition individuals accelerates with energy.
- For biotic resources, there may exist a foraging effort such that yield is maximised and resources are maintained. Due to competition, however, natural selection tends to favour over-consumption.

