Solutions to exercise sheet 4

Sex, Ageing and Foraging Theory

Exercise 1: Competition for renewable resources among relatives

a. Solving for $\hat{n}(x)$ such that

$$\left.\frac{dn}{dt}\right|_{n=\hat{n}(x)} = 0, \tag{1}$$

we obtain equilibrium resource density,

$$\hat{n}(x) = \Big(1 - \frac{n_\mathrm{c} x}{r}\Big) K \ . \tag{2}$$

b. Substituting $\hat{n}(y_\mathrm{r})$ from 1a above into the fitness function (eq. (3) from ex sheet 6 together with the cost eq. (4)), we find that fitness reads as

$$w(y, y_\mathrm{r}, x) = y\Big(1 - \frac{n_\mathrm{c} y_\mathrm{r}}{r}\Big) K - \frac{c_0}{2} y^2 \ . \tag{3}$$

Differentiating this fitness function according to the selection gradient given by eq. (5) in ex sheet 6, we obtain

$$s(x) = \Big(1 - \frac{n_\mathrm{c} x}{r}\Big) K - c_0 x - R_2 \frac{n_\mathrm{c} x}{r} K \ . \tag{4}$$

c. Solving for x^* such that $s(x^*) = 0$, we find that the optimal strategy x^* can be written as

$$x^* = x_\mathrm{MSY} \frac{2Kn_\mathrm{c}}{c_0 r + Kn_\mathrm{c}(1 + R_2)} \ , \tag{5}$$

where

$$x_\mathrm{MSY} = \frac{1}{n_\mathrm{c}} \frac{r}{2} \tag{6}$$

is the foraging effort that lead to maximum sustainable yield. Eq. (5) reveals that the optimal strategy x^* decreases with relatedness, R_2, i.e. individuals evolve to forage less when they do so with relatives. In particular, even in the absence of foraging cost ($c_0 = 0$), individuals avoid over-exploitation when they forage with monozygotic twins (i.e. $x^* = x_\mathrm{MSY}$ when $R_2 = 1$).

Exercise 2: Risk-sensitive foraging

a. See the table below.

Payoff, π_i	Low condition f_{L}	High condition f_{H}
0	0.0	0.0
1	0.9	2.1
2	3.2	3.3

b. In high condition, the fecundity gain from a payoff of 1 to 2 is less than the loss from a payoff of 1 to 0. Selection should therefore favour to avoid risk in high condition individuals (i.e. $x_{\mathrm{H}} \to 0$). By contrast, the fecundity gain from a payoff of 1 to 2 when in low condition is greater than the loss from a payoff of 1 to 0. Selection should therefore lead individuals in low condition to take risk ($x_{\mathrm{L}} \to 1$).

c. The predictions made in 2b above are borne out when running individual based simulations (Fig.1).

d. (i) See Fig. 2 Bottom.

(ii) To adapt the code to take into account normally distributed payoffs (Fig. 2 Top), we need to replace the resource function with the following piece of code:

```
resource = function(xH, xL, env){
        if( env == "low") ri = ifelse(rbinom(1, 1, xL),
                                        rnorm(1,1,1), rnorm(1,1,0.1))
        else ri = ifelse(rbinom(1, 1, xH),
                                        rnorm(1,1,1), rnorm(1,1,0.1))
        ri*(ri > 0)
}
```

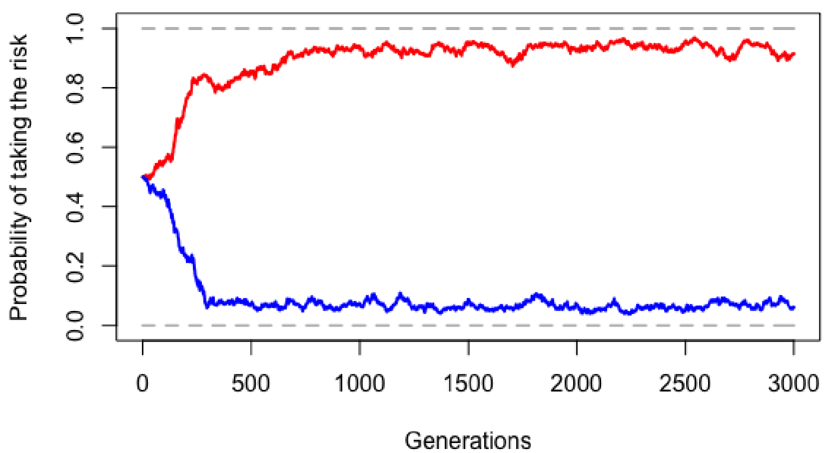

Figure 1: Evolution of the average probabilities of choosing the risk-taking strategy when in low and high condition, x_{L} (in red) and x_{H} (in blue). The population is initially monomorphic for $x_{\mathrm{H}} = x_{\mathrm{L}} = 0.5$.

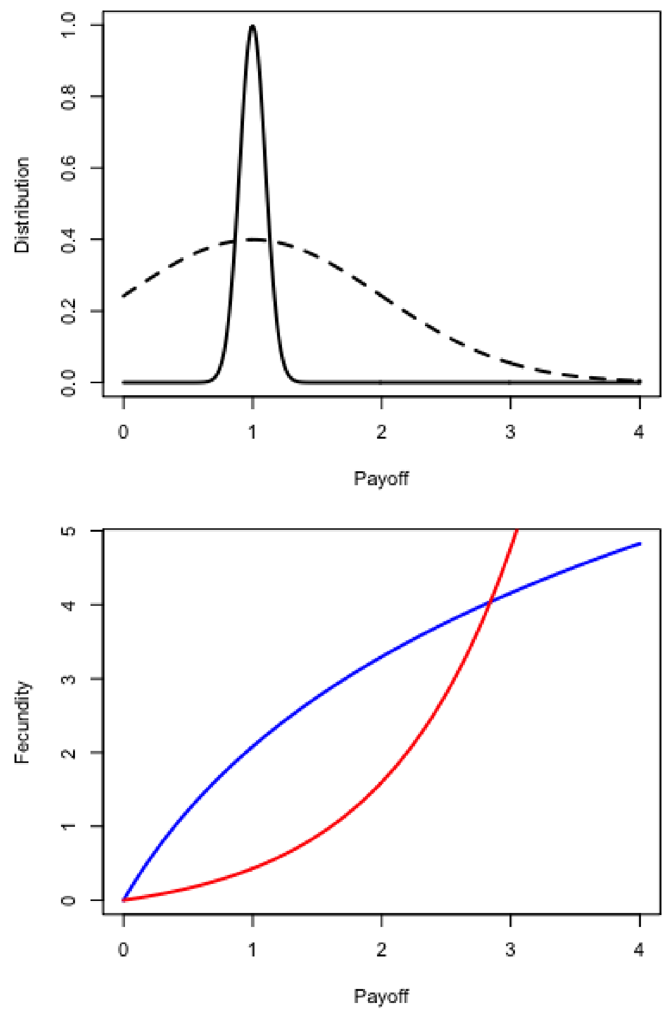

Figure 2: The top plot is the probability of receiving a payoff π_i when foraging with a safe strategy (continuous line) or with a risk-taking strategy (dashed line). Parameters suggested in the Exercise Sheet were employed. The bottom figure is the fecundities in high and low conditions, f_{H} (blue) and f_{L} (red), as a function of the payoff π_i.