

# Exercise sheet 1: Age-structured populations

## Sex, Ageing and Foraging Theory

In this exercise sheet, we model the dynamics of a wild boar population (*Sus scrofa*) using data from the literature. For simplicity, we assume boars cannot live for more than six years, such that the population is structured into a newborn age-class (year 0) and six reproducing age-classes (from year 1 to year 6). Age-specific fecundities and survival are given in Table 1.

Age $a$ (in years)	Fecundity $m_a$	Survival probability $p_a$
0	-	0.8
1	0.57	0.52
2	2.10	0.60
3	4.25	0.71
4	4.25	0.71
5	4.25	0.71
6	4.25	-

Table 1: *Sus scrofa* life-table.

## 1 Leslie Matrix

- Construct the Leslie matrix for this wild-boar population (recall that a Leslie matrix depends on **effective** fecundities,  $f_a$ ).
- Using this matrix, compute the dynamics of the population with R (or another programming language), over ten years starting with  $n_{1,0} = 1000$  individuals of age 1 (and none in the other age classes). Plot the trajectories of the number of individuals in each class predicted by the matrix and comment it briefly (Hint: matrix product is achieved by the `%*%` operator in R).
- Using eigen analysis (tip: use the **eigen()** function in R), work out the growth rate  $\lambda$  of the population, and its stable age distribution  $u$ .
- Compute  $R_0$ . What does it tell you about the fate of the population in the long run?

## 2 Individual based simulations

We provide a R program that allows to track the birth and fate of each individual in an age-structured population, i.e. that allows to perform individual-based simulations (<https://lab-mullon.github.io/SAF>). Download and familiarise yourself with this program.

- Using the provided individual-based simulation program and the information given above, simulate the stochastic dynamics of the wild boar population for ten years, starting with  $n_{1,0} = 1000$  individuals of age 1 (parameter **n0** in the simulation program).
- Plot the simulated population's size as a function of time along with the trajectory predicted from the Leslie matrix (from question 1b above). Do they match?
- Plot the final age distribution of the simulated population and the one predicted by the Leslie matrix on the same graph (from question 1c above). Do they match?

### 3 Density regulation

We now turn to a more realistic scenario where the wild boar population is density-regulated. Specifically, we assume that the establishment probability,  $p_0$ , decreases with the total population size  $N_t = \sum_{a=1}^6 n_{a,t}$  at time  $t$  (Equation 1):

$$p_0(N_t) = \frac{c}{1 + \gamma N_t}, \quad (1)$$

where  $c > 0$  and  $\gamma > 0$  are positive constants.

- Modify the individual based simulation program (from part 2) to incorporate this new assumption (hint: you have to add an argument to the 'DYN' function). Simulate the population for a hundred years, starting with  $n_{1,0} = 1000$  individuals of age 1, with  $c = 0.8$  and  $\gamma = 0.0005$ . How does population size vary over time? Why?
- Construct the Leslie matrix associated with this new model and iterate it over a hundred years starting with  $n_{1,0} = 1000$  individuals of age 1 to model changes in population size. How is population size predicted to vary? Does it match your simulation results (from 3a)?
- Calculate lifetime reproductive success  $R_0$  for each time step in the simulation and plot it over time. How does  $R_0$  change with time and why?