Exercise 6: aggressivity under limited dispersal

Consider a haploid population subdivided into infinitely many patches, each containing exactly $n \geq 2$ adults. Trait $z \in X = [0, 1]$ is the probability of behaving aggressively in a contest, as in Exercise 1.

Individuals repeatedly interact with randomly sampled patchmates. The expected payoff of a focal expressing $z_\bullet$, when its partner expresses z_1, is

$$P(z_\bullet, z_1) = \frac{V}{2} + \frac{V}{2}(z_\bullet - z_1) - Cz_\bullet z_1.$$

Payoff determines fecundity according to

$$f(z_\bullet, z_1) = f_0 + \delta P(z_\bullet, z_1), \qquad 0 < \delta \ll 1,$$

so that payoff has a weak effect on fecundity. Assume fecundity remains positive.

Exercise 6 (continued): life cycle

The population follows this life cycle:

1. Adults interact locally and reproduce.
2. Each offspring disperses to a randomly chosen patch with probability m, or remains in its natal patch with probability $1 - m$.
3. Each adult survives independently with probability s.
4. Juveniles compete for the vacant breeding spots, so that every patch again contains exactly n adults.

Let

$$\mathbf{z}_n = (z_1, \ldots, z_{n-1})$$

denote the phenotypes of the focal individual's patchmates.

Exercise 6 (continued): individual fitness

For an individual i, let $\bar{z}_i = \sum_{j \neq i} z_j/(n-1)$ be mean phenotype among its patchmates. Because $P(z_i, z_j)$ is linear in z_j, expected fecundity across repeated random encounters is $f(z_i, \bar{z}_i)$.

Mean fecundity in the focal patch is $\bar{f}(z_\bullet, \mathbf{z}_n) = \left[f(z_\bullet, \bar{z}_\bullet) + \sum_{j=1}^{n-1} f(z_j, \bar{z}_j)\right]/n$. Because the mutant is rare, immigrants originate from resident patches and have fecundity $f(x, x)$.

a. Show that individual fitness is

$$w(z_\bullet, \mathbf{z}_n, x) = s + (1-s)\left[\frac{(1-m)f(z_\bullet, \bar{z}_\bullet)}{(1-m)\bar{f}(z_\bullet, \mathbf{z}_n) + mf(x,x)} + \frac{mf(z_\bullet, \bar{z}_\bullet)}{f(x,x)}\right].$$

Identify the contributions from adult survival, local recruitment and recruitment after dispersal. Verify that $w(x, (x, \ldots, x), x) = 1$.

Exercise 6 (continued): relatedness

Let r_2° be neutral pairwise relatedness: the probability that a randomly sampled patchmate of a mutant also belongs to its neutral lineage.

Looking one census backwards, show that at equilibrium r_2° satisfies

$$r_2^\circ = s^2 r_2^\circ + \left[2s(1-s)(1-m) + (1-s)^2(1-m)^2\right]\left[\frac{1}{n} + \frac{n-1}{n} r_2^\circ\right].$$

b. Solve the recursion to express r_2° in terms of s, m and n. How does relatedness change with adult survival, dispersal and patch size? Examine the cases $m = 0$, $m = 1$ and $s = 0$.

Exercise 6 (continued): evolution of aggressivity

For a group-structured population, the selection gradient is

$$S(x) = \left.\frac{\partial w(z_\bullet, \mathbf{z}_n, x)}{\partial z_\bullet}\right|_{z_\bullet = z_1 = \cdots = z_{n-1} = x} + (n-1) r_2^\circ \left.\frac{\partial w(z_\bullet, \mathbf{z}_n, x)}{\partial z_1}\right|_{z_\bullet = z_1 = \cdots = z_{n-1} = x}.$$

c. Using the fitness function and relatedness derived above, show that the sign of the selection gradient is given by

$$S(x) \propto \left(\frac{V}{2} - Cx\right) + \kappa^\circ \left(-\frac{V}{2} - Cx\right),$$

where

$$\kappa^\circ = \frac{2s(1-m)}{n(2-m) + s[2 + (n-2)m]}.$$

Find the singular phenotype and determine its convergence stability. How do survival, dispersal and patch size affect evolved aggressivity? Why?

Exercise 7: coordinated hunting under limited dispersal

Return to coordinated hunting example from day 1 (exercise 3). Trait $z \in X = [0, 1]$ is the probability of attempting a joint hunt. The payoff of a focal expressing $z_\bullet$, when its partner expresses z_1, is

$$P(z_\bullet, z_1) = 1 - z_\bullet + 2z_\bullet z_1.$$

The patch life cycle is unchanged from exercise 6.

a. Use the selection gradient to show that the sign of the selection gradient is given by

$$S(x) \propto (-1 + 2x) + \kappa^\circ(2x).$$

Find the singular phenotype and determine its convergence stability. How do adult survival, dispersal and patch size affect the threshold above which coordinated hunting evolves? Why?