Exercise 4: which demographic changes matter most?

Consider a resident population with two classes: juveniles (class 1) and adults (class 2). Under neutrality, its mean matrix is

$$\mathbf{W}^\circ = \begin{pmatrix} 0 & 3/5 \\ 1/2 & 7/10 \end{pmatrix},$$

where rows give offspring class and columns give parent class.

a. Interpret each non-zero entry biologically. Show that the leading eigenvalue of $\mathbf{W}^\circ$ is one. Why must this be true for a resident population at demographic stationarity?

b. Calculate and interpret the class distribution of a neutral lineage $\mathbf{q}^\circ$ and the reproductive values $\mathbf{v}^\circ$.

Exercise 4 (continued)

c. Consider three possible mutants: (i) a mutant whose adult fecundity w_{12} is increased by a small proportion $\varepsilon > 0$; (ii) a mutant whose juvenile maturation w_{21} is increased by the same proportion; (iii) a mutant whose adult survival w_{22} is increased by the same proportion. For each mutant, assume that only the indicated vital rate changes and that all other entries remain unchanged. If w_{ij} increases by a proportion ε, then

$$\rho(y, x) - 1 = \varepsilon v_i^\circ w_{ij}^\circ q_j^\circ + \mathcal{O}(\varepsilon^2).$$

Calculate and compare the initial growth advantage $\rho(y, x) - 1$ of each mutant. Which mutant grows fastest when rare? Why? Explain your answer using the frequency and reproductive value of the parent and offspring classes involved in each vital rate.

Exercise 5: reproduce now or survive to reproduce again?

Consider an organism with at most two breeding opportunities. The evolving trait $z \in X = [0, 1]$ is investment in reproduction during the first opportunity.

The population follows this life cycle:

1. At their first breeding opportunity, individuals produce on average $f_1(z) = f_0 + \beta z$ newborns.
2. They survive to a second breeding opportunity with probability $s_1(z) = s_0(1 - z^2)$.
3. At their second opportunity, individuals produce f_2 newborns and then die.
4. Each newborn recruits into the first breeding class with probability $p_{\mathrm{rec}}(N_t) = 1/(1 + \gamma N_t)$, where N_t is total population size.

Exercise 5 (continued)

Use classes 1 and 2 for individuals at their first and second breeding opportunities.

a. Translate the life cycle into the mutant mean matrix $\mathbf{W}(y, x)$. Remember that rows indicate offspring class, columns indicate parent class, and the resident population is at $\hat{N}(x)$.

The basic reproductive number $R_0(y, x)$ is the expected number of recruited offspring produced over the lifetime of one mutant entering the first breeding class.

Show that

$$R_0(y, x) = p_{\mathrm{rec}}(\hat{N}(x)) \left[f_0 + \beta y + s_0(1 - y^2) f_2 \right].$$

.

Exercise 5 (continued)

b. Since $\rho(y,x) - 1$ and $R_0(y,x) - 1$ have the same sign, we can use R_0 to characterise gradual evolution. The lifetime-reproduction gradient satisfies

$$\widetilde{S}(x) = \left.\frac{\partial R_0(y,x)}{\partial y}\right|_{y=x} \propto \left.\frac{\partial}{\partial y}\left[f_0 + \beta y + s_0(1-y^2)f_2\right]\right|_{y=x}.$$

- Use $\widetilde{S}(x)$ to find the singular phenotype x^*. When is it inside $[0,1]$, and is it convergence stable?
- How does survival to the second breeding opportunity, s_0, affect x^*? When does evolution lead to complete investment in the first breeding opportunity, $x = 1$?
- Explain why increasing mortality can favour the evolution of a "live fast, die young" life history.

Exercise 5 (continued)

c. A persistent resident population at demographic stationarity satisfies $R_0(x, x) = 1$. Use this condition to find $\hat{N}(x)$.

As long as x^* is inside $[0, 1]$, determine how the evolved population size $\hat{N}(x^*)$ changes when survival s_0 decreases. Does the evolution of greater current reproduction compensate for the loss of future reproduction?