Exercise 1 (continued): selection on aggressivity

Recall the results from this morning:

$$P(y,x) = \frac{V}{2}(1-x) + y\left(\frac{V}{2} - Cx\right),$$

$$f(y,x) = f_0 + \delta P(y,x), \qquad \rho(y,x) = s + (1-s)\frac{f(y,x)}{f(x,x)}.$$

d. Derive the selection gradient

$$S(x) = \left.\frac{\partial \rho(y,x)}{\partial y}\right|_{y=x}.$$

Which trait changes are favoured when resident aggressivity is low? What changes when resident aggressivity is high?

Exercise 1 (continued)

e. Characterise gradual evolution.

- Find the singular strategy x^*. When is it inside $X = [0, 1]$?
- Calculate $S'(x^*)$ and determine whether x^* is convergence stable.
- How do resource value V and fighting cost C affect x^*? Why?
- How does adult survival s affect the difference $\rho(y, x) - 1$ between mutant and resident invasion fitness? Interpret the result biologically.

Exercise 1 (continued)

f. Relate trait evolution to changes in population size. Recall that

$$\hat{N}(x) = \frac{1}{\gamma}\left[\frac{f(x,x)}{1-s} - 1\right].$$

- Write $\hat{N}(x)$ explicitly and determine how equilibrium population size changes with resident aggressivity x using $\hat{N}'(x)$.
- Suppose that the population initially expresses $0 < x < x^*$. Does selection favour an increase or a decrease in aggressivity? What happens to population size during evolution?
- Does natural selection maximise population size in this model? Explain the result biologically.

Exercise 3: cooperative hunting

Some predators can hunt alone for small prey or coordinate with a partner to capture larger prey. A joint hunt is profitable when both individuals participate, but fails when one attempts to cooperate and the other hunts alone.

The evolving trait $z \in [0, 1]$ is the probability of attempting a cooperative hunt. The resident expresses x and a rare mutant expresses y.

Can a cooperative strategy that supports a larger population nevertheless fail to evolve?

Exercise 3 (continued)

The population follows this life cycle:

1. Adults experience many foraging bouts and are randomly paired during each bout. An individual expressing phenotype $z \in [0, 1]$ attempts to hunt cooperatively with probability z, and hunts alone otherwise. (i) If both individuals cooperate, they hunt successfully and each obtains two units of food. (ii) If the focal individual hunts alone, it obtains one unit of food, irrespective of its partner's behaviour. (iii) If the focal individual cooperates but its partner hunts alone, the cooperative hunt fails and the focal individual obtains no food.
2. Food accumulated across foraging bouts determines fecundity: $f(y, x) = 2 + P(y, x)$, where $P(y, x)$ is the expected food intake of an individual expressing phenotype y in a population monomorphic for x.

Exercise 3 (continued)

3. Each offspring recruits independently with probability $1/(1 + \gamma N_t)$, where $\gamma > 0$.
4. Adults survive independently with probability $s \in [0, 1)$.

a. Consider a rare mutant expressing phenotype y in a resident population expressing x.

- Derive its expected food intake $P(y, x)$ by considering all possible combinations of mutant and resident behaviour.
- Write the recursion for the resident population and find its positive equilibrium $\hat{N}(x)$.
- Derive invasion fitness $\rho(y, x)$ and verify that $\rho(x, x) = 1$.

Exercise 3 (continued)

b. Characterise phenotypic evolution.

- Derive $S(x)$ and find the interior singular strategy x^*.
- Calculate $S'(x^*)$ and show that x^* is not convergence stable.
- Plot $S(x)$ over $x \in [0, 1]$.

c . Interpret x^* as a threshold between two evolutionary outcomes: completely solitary foraging and completely cooperative hunting.

- Calculate the population sizes supported by completely solitary foraging and completely cooperative hunting.
- Why does the fact that cooperative hunting supports more individuals not guarantee that it evolves?
- What biological feature of the interaction generates evolutionary bistability?