Exercise 5: the evolution of aggressivity under limited dispersal

We now revisit the model of aggressivity evolution from day 1, but under limited dispersal.

A priori: how do you expect limited dispersal to affect the evolution of aggressivity?

Assume the population is subdivided into a large number of patches of size $n = 2$, with reproduction and death following a Moran process. Fecundity is determined by the payoff from repeated local interactions over a resource of value V.

The evolving trait $z \in [0, 1]$ is the probability of behaving aggressively in a contest.

Exercise 5: life cycle under limited dispersal

Specifically, the life cycle proceeds as follows:

1. Individuals engage in repeated pairwise contests within patches. In each contest, they play either aggressive or docile with payoffs per interaction:
   - Docile vs. docile: share V equally.
   - Aggressive vs. docile: aggressive gets all.
   - Aggressive vs. aggressive: one wins V with probability $1/2$, both pay cost $C > 0$.
2. Individuals produce a large number of offspring. A focal individual with trait $z_\bullet$ with a neighbour with trait z_1 has fecundity $f(z_\bullet, z_1) = 1 + \delta\pi(z_\bullet, z_1)$ is the average payoff received during contests and where $\delta > 0$ is a parameter that tunes the strength of selection.
3. Offspring either disperse with probability m (in which case they land in a uniformly chosen patch, i.e. island model of dispersal) or stay with probability $1 - m$.
4. One adult per patch dies at random; offspring compete locally for the vacant spot.

Exercise 5 (continued)

a. Write the individual fitness function $w(z_\bullet, z_1)$ of a focal individual with trait $z_\bullet$, paired with a neighbour with trait z_1, when the rest of the population is resident at trait x.

b. Use $w(z_\bullet, z_1)$ and the relatedness coefficient r_2° (as derived during the lecture) to compute the selection gradient $S(x)$. Identify the singular strategy x^*, and assess its convergence stability (make use of the fact that δ is small to make the algebra easier).

What is the effect of limited dispersal (i.e. small m) on the singular strategy ?

c. Using the fact that under the Moran model when traits affect fecundity only : $\partial r_2(y, x)/(\partial y)|_{y=x=x^*} = 0$ holds, calculate the coefficient of disruptive selection $H(x^*)$.

How does limited dispersal affect disruptive selection? Why?

Exercise 6: the evolution of altruism for survival

We saw that altruism can evolve when individuals reduce their fecundity to increase that of their neighbours under limited dispersal.

Here, we ask: does selection on altruism change when it affects survival instead of fecundity?

Consider a population subdivided into patches of size $n = 2$, with reproduction and death following a Moran process and life-cycle:

1. Individuals interact within patches, providing a survival benefit b to their partner at a cost c to themselves.
2. Individuals reproduce with fixed fecundity.
3. Offspring disperse with probability m, or remain with probability $1 - m$.
4. One adult per patch dies. The probability of dying is determined by payoff. Offspring compete locally for the vacant spot.

Exercise 6 (continued)

The evolving trait $z \in [0, 1]$ is the amount invested into the partner's survival at a cost to oneself.

If a focal individual has trait $z_\bullet$ and its neighbour z_1, their respective survival payoffs are:

$$P_\bullet = 1 + bz_1 - cz_\bullet, \quad P_1 = 1 + bz_\bullet - cz_1,$$

so that the probability that the focal survives is:

$$\frac{P_\bullet}{P_\bullet + P_1}.$$

Exercise 6 (continued)

a. Write the individual fitness $w(z_\bullet, z_1)$ of a focal in a resident population.

b. Using $w(z_\bullet, z_1)$ and relatedness r_2°, compute the selection gradient $S(x)$. What is the nature of directional selection on altruism in this case (positive, negative)? Why?

c. Now suppose that altruism still reduces survival payoff $(1 - cz_\bullet)$, but increases neighbour fecundity to $1 + bz_1$. Does this change directional selection on altruism?