Exercise 1: survival–fecundity trade-off

All organisms face trade-offs due to limited resources. One classical trade-off occurs between investment into survival and into reproduction, both major components of fitness.

Can such a trade-off favour diversification, with different types specialising in either survival or reproduction?

Consider a well-mixed population of constant size N with the following life cycle:

1. Adults produce a large number of offspring.
2. Adults survive with a certain probability; otherwise they die.
3. Offspring compete randomly for the vacant breeding spots.

The evolving trait z increases the survival probability, which is $s_{\max} z$ where $0 < s_{\max} < 1$ is a parameter for the maximum survival. Fecundity is a decreasing function of survival, $f(z) > 0$, with $f'(z) < 0$.

Exercise 1 (continued)

a. Define the invasion fitness $\rho(y, x)$ of a rare mutant trait y in a resident population expressing trait x.

b. Let $f(z) = f_{\max}(1 - z^2)$ where $f_{\max} > 0$ is a parameter. For this specific trade-off:

- Find the singular strategy x^*.
- Determine whether x^* is convergence stable.
- Determine whether x^* is uninvadable or whether evolutionary branching is possible.

c. For an arbitrary smooth decreasing function $f(z)$, can evolutionary branching occur? (hint: recall the necessary condition of negative trait dependent selection for polymorphism)

Exercise 2: the evolution of aggressivity

Many animals compete directly over resources such as mates, food, or nesting sites. In such contests, individuals may differ in aggressivity. When a more aggressive individual meets a less aggressive one, the latter tends to retreat. But when two aggressive individuals meet, the contest escalates into a costly fight.

Can such interactions favour social polymorphism, with both aggressive and docile individuals coexisting?

Exercise 2: the evolution of aggressivity

Consider a well-mixed population of constant size N, with the following life cycle:

1. Individuals engage in repeated pairwise contests over a resource of value V (e.g. in calories).
2. In each contest, an individual expresses either aggressive or docile behaviour (with probability z and $1 - z$, resp.).
3. When two docile individuals meet, they share the resource equally. When a docile and an aggressive individual meet, the aggressive one obtains the full resource. When two aggressive individuals meet, one wins the resource with probability $1/2$, but both pay a cost $C > 0$ (e.g. in calories) from fighting.
4. Individuals reproduce with fecundity that increases with the resources accumulated minus any fighting costs, and then die.
5. Offspring compete randomly for the vacant breeding spots.

The evolving trait $z \in [0, 1]$ is the probability of behaving aggressively during a contest.

Exercise 2 (continued)

a. Derive the expected payoff $\pi(y, x)$ of a mutant with aggressivity probability y in a resident population with trait x. Use this to characterise its fecundity $f(y, x) = 1 + \delta\pi(y, x)$ where $\delta > 0$ is a parameter that tunes the strength of selection.

b. Define the invasion fitness $\rho(y, x)$ of a rare mutant in a resident population. Assuming weak selection (i.e. that δ is small) :

- Find the singular strategy x^* and determine whether it's convergence stable.
- Determine whether x^* is uninvadable, or whether evolutionary branching is possible.

c. Suppose expressing a mix of behaviours carries a cost of flexibility:

$$f(y, x) = 1 + \delta[\pi(y, x) - c_\mathrm{f}\, y(1 - y)],$$

where $c_\mathrm{f} > 0$ is a flexibility cost parameter. What is the effect of this cost on the possibility of evolutionary branching?